Preprint typeset in JHEP style - HYPER VERSION

LMU-ASC 54/07

$k-$Essence, superluminal propagation, causality and emergent geometry

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ABSTRACT: The k-*essence* theories admit in general the superluminal propagation of the perturbations on classical backgrounds. We show that in spite of the superluminal propagation the causal paradoxes do not arise in these theories and in this respect they are not less safe than General Relativity.

Contents

1. Introduction

Over the past years, spontaneous breaking of the Lorentz invariance and questions related to this issue, such as superluminal propagation of perturbations in nontrivial backgrounds, attracted a renewed interest among physicists. One of the basic questions here is whether the theories allowing the superluminal velocities possess internal inconsistencies and, in particular, inevitably lead to the causality violation namely to the appearance of the closed causal curves (CCCs). Concerning this issue there exist two contradicting each other points of view. Some authors (see, for instance, [1, 2, 3, 4, 5, 6, 7, 8]) argue that the subluminal propagation condition should a priori be imposed to make the theory physically acceptable. For example, in [1] on the P. 60 the authors introduce the "*Postulate of Local Causality*" which excludes the superluminal velocities from the very beginning. The requirement of subluminality is sometimes used to impose rather strong restrictions on the form of the admissible Lagrangians for the vector and higher spin fields [5] and gravity modifications [7]. The effective field theories (EFT) allowing the superluminal propagation were considered in [8], where it was argued that in such theories global causality and analyticity of the S-matrix may be easily violated. The main conclusion of [8] is not favorable for the theories with superluminal propagation. In particular the authors claim that the UV-completion of such theories must be very nontrivial if it exists at all (for a different attitude see [9, 10]).

An open minded opinion concerning the superluminal propagation is expressed in [11], where one argues that the proper change of the chronological ordering of spacetime in non-linear field theory with superluminal propagation allows us to avoid the causal paradoxes.

Recently, in the literature were discussed several cases in which faster-than-light propagation arises in a rather natural way. In particular we would like to mention the noncommutative solitons [12], Einstein aether waves [13], "superluminal" photons in the Drummond-Hathrell effect [14, 15] and in the Scharnhorst effect [16, 17] [1]. These last two phenomena are due to the vacuum polarization i.e. higher-order QED corrections. It was argued that this superluminal propagation leads to the causal paradoxes in the gedanken experiment involving either two black holes [18] or two pairs of Casimir plates [19] moving with the high relative velocities. To avoid the appearance of the closed causal curved in such experiments the authors of [19] invoked the *Chronology Protection Conjecture* [20] and showed that the photons in the Scharnhorst effect causally propagate in effective metric different from the Minkowski one.

Note that the superluminal propagation cannot be the sole reason for the appearance of the closed causal curves. There are numerous examples of spacetimes in General Relativity, where the *"Postulate of Local Causality"* is satisfied and, nevertheless, the closed causal curves are present (see [21, 22, 23, 24, 25]). Therefore an interesting question arises whether the superluminal propagation leads to additional problems related with causality compared to the situation in General Relativity.

In these paper we will consider the k-*essence* fields [26, 27, 28, 29] and show that contrary to the claim of [2, 30] the causality is not violated in generic k-*essence* models with superluminal propagation (similar attitude was advocated in [31, 32, 33]). In this sense, in spite of the presence of superluminal signals on nontrivial backgrounds, the k-*essence* theories are not less safe and legitimate than General Relativity.

The paper is organized as follows. In Section 2 we discuss the equation of motion for k-*essence* and derive generally covariant action for perturbations for an arbitrary k-*essence* background.

General aspects of causality and propagation of perturbations on a nontrivial background, determining the "*new aether*", are discussed in Section 3. In particular, we prove that no causal paradoxes arise in the cases studied in our previous works [27, 28, 26] and [29].

Section 4 is devoted to the Cauchy problem for k-*essence* equation of motion. We investigate under which restrictions on the initial conditions the Cauchy problem is well posed.

In Section 5 we study the Cauchy problem for small perturbations in the "*new aether*" rest frame and in the fast moving spacecraft.

Section 6 is devoted to the *Chronology Protection Conjecture,* which is used to avoid the CCCs in gedanken experiments considered in [8].

In Section 7 we discuss the universal role of the gravitational metric. Namely, we show that for the physically justified k-*essence* theories the boundary of the smooth field configuration localized in Minkowski vacuum, can propagate only with the speed not exceeding the speed of light. In agreement with this result we derive that exact solitary waves in purely kinetic k-*essence* propagate in vacuum with the speed of light.

Our main conclusions are summarized in Section 8.

All derivations of more technical nature the reader can find in Appendices. In Appendix A we derive characteristics of the equation of motion and discuss local causality. Appendix B is devoted to the derivation of the generally covariant action for perturbations. In Appendix C we show how the action derived in Appendix B is related to the action for cosmological perturbations from [28, 34].

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[1] In this paper under "superluminal" we always mean "faster than light in usual QED vacuum in unbounded empty space". To avoid confusion one could say that photons propagate faster than gravitons in the Scharnhorst effect. When this paper was in the final stage of preparation the superluminal wave-front velocity in these effects was putted under question [10].

In Appendix D we consider the connection between k-*essence* and hydrodynamics. The derivation of Green functions is given in Appendix E.

2. Equations of motion and emergent geometry

Let us consider the k-*essence* scalar field ϕ with the action:

$$S_\phi = \int d^4x\sqrt{-g}\mathcal{L}\left(X, \phi\right), \tag{2.1}$$

where

$$X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi,$$

is the canonical kinetic term and by ∇_μ we always denote the covariant derivative associated with metric $g_{\mu\nu}$. We would like to stress that this action is explicitly generally covariant and Lorentz invariant. The variation of action (2.1) with respect to $g_{\mu\nu}$ gives us the following energy-momentum tensor for the scalar field:

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \mathcal{L}_{,X}\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\mathcal{L}, \tag{2.2}$$

where $\left(...\right)_{,X}$ is the partial derivative with respect to X. The Null Energy Condition (NEC) $T_{\mu\nu}n^\mu n^\nu \geq 0$ (where n^μ is a null vector: $g_{\mu\nu}n^\mu n^\nu = 0$) is satisfied provided $\mathcal{L}_{,X} \geq 0$. Because violation of this condition would imply the unbounded from below Hamiltonian and hence signifies the inherent instability of the system [35] we consider only the theories with $\mathcal{L}_{,X} \geq 0$.

The equation of motion for the scalar field is obtained by variation of action (2.1) with respect to ϕ,

$$-\frac{\delta S}{\delta\phi} = \tilde{G}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + 2X\mathcal{L}_{,X\phi} - \mathcal{L}_{,\phi} = 0, \tag{2.3}$$

where the "effective" metric is given by

$$\tilde{G}^{\mu\nu}\left(\phi, \nabla\phi\right) \equiv \mathcal{L}_{,X}g^{\mu\nu} + \mathcal{L}_{,XX}\nabla^\mu\phi\nabla^\nu\phi. \tag{2.4}$$

This second order differential equation is hyperbolic (that is, $\tilde{G}^{\mu\nu}$ has the Lorentzian signature) and hence describes the time evolution of the system provided [6, 36, 33]

$$1 + 2X\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} > 0. \tag{2.5}$$

When this condition holds everywhere the effective metric $\tilde{G}^{\mu\nu}$ determines the characteristics (cone of influence) for k-*essence*, see e.g. [36, 33, 37, 38]. For *nontrivial configurations* of k-*essence* field $\partial_\mu\phi \neq 0$ and the metric $\tilde{G}^{\mu\nu}$ is generally not conformally equivalent to $g^{\mu\nu}$; hence in this case the characteristics do not coincide with those ones for canonical scalar field the Lagrangian of which depends linearly on the kinetic term X. In turn, the characteristics determine the *local causal structure* of the space time in every point of the manifold. Hence, the *local causal structure* for the k-*essence* field is generically different from those one defined by metric $g_{\mu\nu}$ (see Appendix A for details). For the coupled system of equations for the gravitational field and k-*essence* the Cauchy problem is well posed only if the initial conditions are posed on the hypersurface which is spacelike with respect to both metrics: $g^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ (see P. 251 of Ref. [39] and Refs. [40, 36, 41] for details). We postpone the detailed discussion of this issue until Section 4 and now we turn to the behavior of small perturbations on a given background. With this purpose it is convenient to introduce the function

$$c_s^2 \equiv \left(1 + 2X\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)^{-1}, \tag{2.6}$$

which for the case $X > 0$ plays the role of "speed of sound" for small perturbations [28] propagating in the preferred reference frame, where the background is at rest. It is well known that in the case under consideration there exists an equivalent hydrodynamic description of the system (see Appendix D) and the hyperbolicity condition (2.5) is equivalent to the requirement of the hydrodynamic stability $c_s^2 > 0$.

The Leray's theorem (see P. 251 of Ref. [39] and Ref. [40]) states that the perturbations π on given background $\phi_0(x)$ propagate causally in metric $\tilde{G}^{\mu\nu}(\phi_0, \nabla\phi_0)$. In Appendix B we show that neglecting the metric perturbations $\delta g_{\mu\nu}$, induced by π, one can rewrite the equation of motion for the scalar field perturbations in the following form

$$\frac{1}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}G^{\mu\nu}\partial_\nu\pi\right) + M_{\rm eff}^2\pi = 0, \tag{2.7}$$

here we denote

$$G^{\mu\nu} \equiv \frac{c_s}{\mathcal{L}_{,X}^2}\tilde{G}^{\mu\nu}, \quad \sqrt{-G} \equiv \sqrt{-\det G_{\mu\nu}^{-1}} \quad \text{where} \quad G_{\mu\lambda}^{-1}G^{\lambda\nu} = \delta_\mu^\nu, \tag{2.8}$$

and

$$M_{\rm eff}^2 \equiv \frac{c_s}{\mathcal{L}_{,X}^2}\left(2X\mathcal{L}_{,X\phi\phi} - \mathcal{L}_{,\phi\phi} + \frac{\partial\tilde{G}^{\mu\nu}}{\partial\phi}\nabla_\mu\nabla_\nu\phi_0\right). \tag{2.9}$$

Note that the metric $G^{\mu\nu}$ is conformally equivalent to $\tilde{G}^{\mu\nu}$ and hence describes the same causal structure as it must be. The equation for the perturbations has exactly the same form as equation for the massive Klein-Gordon field in the curved spacetime. Therefore the metric $G^{\mu\nu}$ describes the "*emergent*" or "*analogue*" spacetime where the perturbations live. In particular this means that the action for perturbations

$$S_\pi = \frac{1}{2}\int d^4x\sqrt{-G}\left[G^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - M_{\rm eff}^2\pi^2\right], \tag{2.10}$$

and the equation of motion (2.7) are generally covariant in the geometry $G^{\mu\nu}$. Introducing the covariant derivatives D_μ associated with metric $G^{\mu\nu}$ ($D_\mu G^{\alpha\beta} = 0$), equation (2.7) becomes

$$G^{\mu\nu}D_\mu D_\nu\pi + M_{\rm eff}^2\pi = 0. \tag{2.11}$$

Using the inverse to $G^{\mu\nu}$ matrix

$$G_{\mu\nu}^{-1} = \frac{\mathcal{L}_{,X}}{c_s}\left[g_{\mu\nu} - c_s^2\left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)\nabla_\mu\phi_0\nabla_\nu\phi_0\right], \tag{2.12}$$

one can define the "emergent" interval

$$dS^2 \equiv G_{\mu\nu}^{-1}dx^\mu dx^\nu, \tag{2.13}$$

which determines the influence cone for small perturbations of k-*essence* on a given background[2]. This influence cone is larger than those one determined by the metric $g_{\mu\nu}$, provided $\mathcal{L}_{,XX}/\mathcal{L}_{,X} < 0$ [6, 36, 33, 38], and the superluminal propagation of small perturbations becomes possible (see Appendix A). At first glance it looks like the theory under consideration has emergent bimetric structure. However, this theory is inherently different from the bimetric theories of gravity [42] because the emergent metric refers only to the perturbations of k-*essence* and is due to the non-linearity of the theory, while in the bimetric gravity theories both metrics have fundamental origin and are on the same footing.

[2]Note that in order to avoid confusion we will be raising and lowering the indices of tensors by gravitational metric $g^{\mu\nu}$ ($g_{\mu\nu}$) throughout the paper.

The derived above form of the action and of the equation of motion for perturbations is very useful. In particular, it simplifies the stability analysis of the background with respect to the perturbations of arbitrary wavelengths, while the hyperbolicity condition (2.5) guarantees this stability only with respect to the short-wavelength perturbations.

It is important to mention that besides of the usual hyperbolicity condition (2.5) one has to require that $\mathcal{L}_{,X}$ is nowhere vanishes or becomes infinite. The points where $\mathcal{L}_{,X}$ vanishes or diverges, generally correspond to the singularities of the emergent geometry. It follows from equations (2.8) and (2.12) that these singularities are of the true nature and cannot be avoided by the change of the coordinate system. Therefore one can argue that before the singularities are formed the curvature of the emergent spacetime becomes large enough for efficient quantum production of the k-*essence* perturbations which will destroy the classical background and therefore $\mathcal{L}_{,X}$ cannot dynamically change its sign. Hence, if one assumes that at some moment of time the k-*essence* satisfies the null energy condition, that is, $\mathcal{L}_{,X} > 0$ everywhere in the space (or $\varepsilon + p > 0$ in hydrodynamical language; see Appendix D) then this condition can be violated only if one finds the way to pass through the singularity in the emergent geometry with taking into account the quantum production of the perturbations. This doubts the possibility of the smooth crossing of the equation of state $w = -1$ and puts under question recently suggested models of the bouncing universe ([43]). The statements above generalize the results obtained in [44] and re-derived later in different ways in [45] in cosmological context.

In deriving (2.10) and (2.11) we have assumed that the k-*essence* is sub-dominant component in producing the gravitational field and consequently have neglected the metric perturbations induced by the scalar field. In particular the formalism developed is applicable for accretion of a *test* scalar field onto black hole [29]. For k-*essence* dark energy [26] action (2.10) can be used only when k-*essence* is a small fraction of the total energy density of the universe, in particular, this action is applicable during the stage when the speed of sound of a successful k-*essence* has to be larger than the speed of light [2, 32]. During k-*inflation* [46, 47, 27] or DBI inflation [48] the geometry $g_{\mu\nu}$ is determined by the scalar field itself and therefore the induced scalar metric perturbations are of the same order of magnitude as the perturbations of the scalar field. For this case the action for cosmological perturbations was derived in [28], see also [34]. We have shown in Appendix C that the correct action for perturbations in k-*inflation* has, however, the same structure of the kinetic terms as (2.10) or, in other words, the perturbations live in the same emergent spacetime with geometry $G^{\mu\nu}$. One can expect therefore that this emergent geometry $G^{\mu\nu}$ has a much broader range of applicability and determines the causal structure for perturbations also in the case of other backgrounds, where one cannot neglect the induced metric perturbations.

If the hyperbolicity condition (2.5) is satisfied, then at any given point of spacetime the metric $G^{-1}_{\mu\nu}$ can always be brought to the canonical Minkowski form diag $(1, -1, -1, -1)$ by the appropriate coordinate transformation. However, the quadratic forms $g_{\mu\nu}$ and $G^{-1}_{\mu\nu}$ are not positively defined and therefore for a general background there exist no coordinate system where they are both simultaneously diagonal. In some cases both metrics can be nevertheless simultaneously diagonalized at a given point, so that, e.g. gravitational metric $g_{\mu\nu}$ is equal Minkowski metric and the induced metric $G^{-1}_{\mu\nu}$ is proportional to diag $\left(c_s^2, -1, -1, -1\right)$, where c_s is the speed of sound (2.6). For instance, in isotropic homogeneous universe both metrics are always diagonal in the Friedmann coordinate frame.

We conclude this section with the following interesting observation. The effective metric (2.12) can be expressed through the energy momentum tensor (2.2) as

$$G^{-1}_{\mu\nu} = \alpha g_{\mu\nu} + \beta T_{\mu\nu} \tag{2.14}$$

where

$$\alpha = \frac{\mathcal{L}_{,X}}{c_s} - \mathcal{L} c_s \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} \quad \text{and} \quad \beta = -c_s \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}.$$

As we have pointed out the cosmological perturbations propagate in $G^{-1}_{\mu\nu}$ even if the background field determines the dynamics of the universe. In this case the energy momentum tensor for the scalar filed satisfies the Einstein equations and eventually we can rewrite the effective metric in the following form

$$G^{-1}_{\mu\nu} = \left(\alpha - \frac{\beta}{2} R\right) g_{\mu\nu} + \beta R_{\mu\nu}. \tag{2.15}$$

This looks very similar to the "metric redefinition" $g_{\mu\nu} \leftrightarrow G^{-1}_{\mu\nu}$ in string theory where the quadratic in curvature terms in the effective action are fixed only up to "metric redefinition" (2.15) see e.g. [49]. The "metric redefinition" does not change the light cone and hence the *local causality* only in the Ricci flat $R_{\mu\nu} = 0$ spacetimes. However, neither in the matter dominated universe nor during inflation the *local causals structures* determined by $g_{\mu\nu}$ and $G^{-1}_{\mu\nu}$ are equivalent.

3. Causality on nontrivial backgrounds

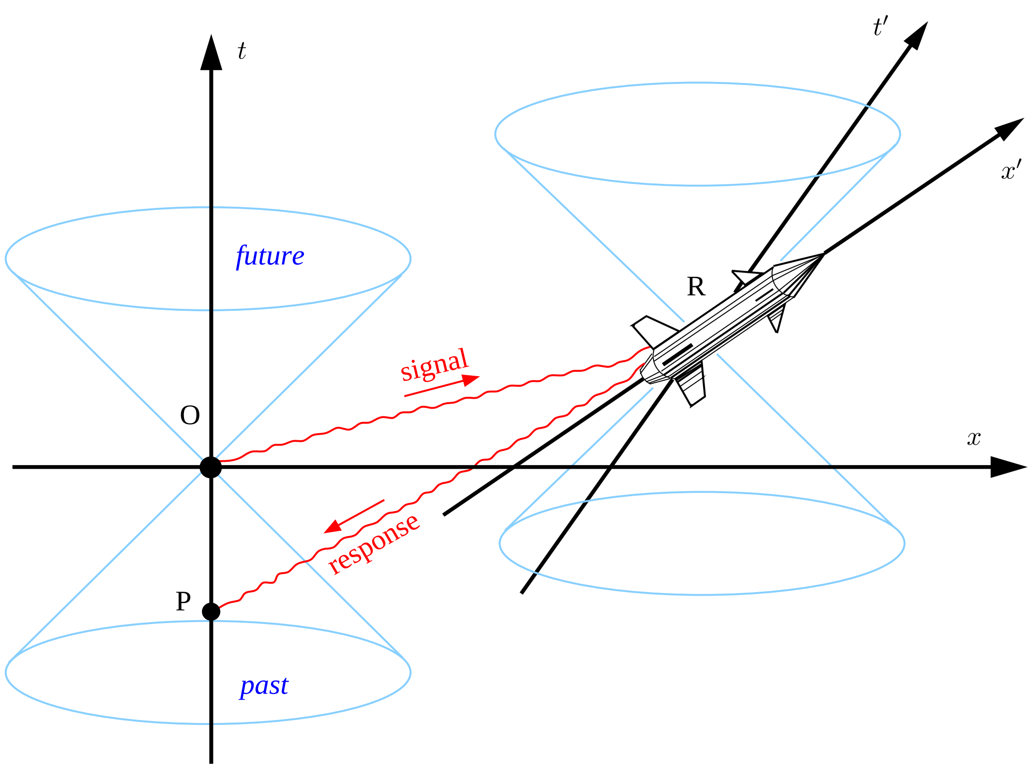

Figure 1: This figure represents the causal paradox constructed using *tachyons*. Someone living along the worldline $x = 0$ sends a tachyon signal to the astronaut in a fast moving spacecraft, OR. In the spacecraft frame (x', t'), the astronaut sends a tachyon signal back, RP. The signal RP propagates in the direction of growing t' as it is seen by the astronaut, however it travels *"back in time"* in the rest frame. Thus it is possible to send a message back in the own past.

In this section we discuss the causality issue for superluminal propagation of perturbations on some nontrivial backgrounds, in particular, in Minkowski spacetime with the scalar field, in Friedmann universe and for black hole surrounded by the accreting scalar field.

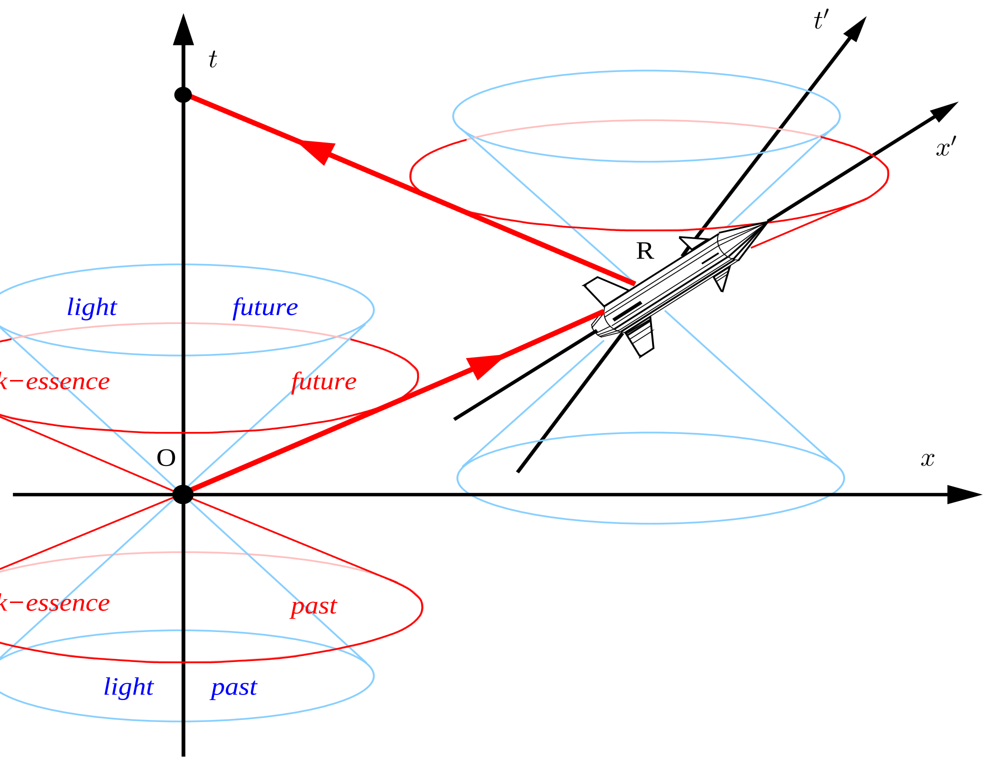

Figure 2: The causality paradox is avoided when superluminal signals propagate in the background which breaks the Lorentz symmetry (compare with Fig. 1). The observers cannot send a message to themselves in the past.

First, we would like to recall a well-known paradox sometimes called "tachyonic anti-telephone" [50] arising in the presence of the superluminal hypothetical particles *tachyons* possessing unbounded velocity $c_{tachyon} > 1$. In this case we could send a message to our own past. Indeed, let us consider some observer, who is at rest at $x = 0$ with respect to the reference frame (x, t) and sends along OR a tachyon signal to an astronaut in the spacecraft R (see Fig. 1). In turn, after receiving this signal, the astronaut communicates back sending the tachyon signal, RP. As this signal propagates the astronaut proper time t' grows. However, if the speed of the spacecraft is larger than $1/c_{tachyon}$, then the signal RP propagates *backward in time* in the original rest frame of the observer. Thus, the observers can in principle send information from "their future" to "their past". It is clear that such situation is unacceptable from the physical point of view.

Now let us turn to the case of the Minkowski space-time filled with the scalar field, which allows the "superluminal" propagation of perturbations in its background. For simplicity we consider a homogeneous time dependant field $\phi_0(t)$. Its "velocity" $\partial_\mu \phi$ is directed along the timelike vector, $u^\mu = (1, 0, 0, 0)$. Why does the paradox above not arise here? This is because the superluminal propagation of the signals is possible only in the presence of nontrivial background of scalar field which serves as the *aether* for sonic perturbations. The *aether* selects the preferred reference frame and clearly the equation of motion for acoustic perturbations is not invariant under the Lorentz transformations unless $c_s = 1$. In the moving frame of the astronaut the equation for perturbations has more complicated form than in the rest frame and the analysis of its solutions is more involved. However, keeping in mind that k-*essence* signals propagate along the characteristics which are coordinate independent hypersurfaces in the spacetime we can study the propagation of sonic perturbations, caused by the astronaut, in the rest frame of the aether and easily find that

the signal propagates always *forward in time* in this frame (see Fig. 2). Hence no closed causal curves can arise here.

We would like to make a remark concerning the notion of "future-" and "past" directed signals. It was argued in [30] that in order to have no CCCs for the k-*essence* during the "superluminal" stage, "...the observers travelling at high speeds with respect to the cosmological frame must send signals backwards in their time for some specific direction". One should remember, however, that the notion of past and future is determined by the past and future cones in the spacetime and has nothing to do with a particular choice of coordinates. Thus, the signals, which are future-directed in the rest-frame remain the future-directed also in a fast-moving spacecraft, in spite of the fact that this would correspond to the decreasing time coordinate t'. As we show in Section 5, the confusion arises because of a poor choice of coordinates, when decreasing t' correspond to future-directed signals and vice versa. The example shown in Fig. 4 illustrates this point: one can see that even without involving superluminal signals, an increasing coordinate time does not always imply the future direction.

Another potentially confusing issue is related to the question which particular velocity must be associated with the speed of signal propagation, namely, phase, group or front velocity. For example, in [30] an acausal paradox is designed using different superluminal group velocities for different wavenumbers. One should remember, however, that neither group nor phase velocities have any direct relation with the causal structure of the spacetime. Indeed the characteristic surfaces of the partial differential equations describe the propagation of the wavefront. This front velocity coincides with the phase velocity only in the limit of the short wavelength perturbations. Generally the wavefront corresponds to the discontinuity of the second derivatives and therefore it moves "off-shell" (a more detailed discussion can be found in e.g. [9]). The group velocity can be less or even larger than the wavefront velocity. One can recall the simple examples of the canonical free scalar field theories: for normal scalar fields the mass squared, $m^2 > 0$, is positive and the phase velocity is larger than c while the group velocity is smaller than c; on the other hand for tachyons ($m^2 < 0$) the situation is opposite. Thus, if the group velocity were the speed of the signal transfer, one could easily build the time-machine similar to those described in [30] using canonical scalar field with negative mass squared, $m^2 < 0$. This, however, is impossible because the causal structure in both cases ($m^2 > 0$ and $m^2 < 0$) is governed by the same *light* cones. Finally we would like to mention that the faster-than-light group velocity has been already measured in the experiment [51].

To prove the absence of the closed causal curves (CCC) in those known situations where the superluminal propagation is possible, we use the theorem from Ref. [39] (see p. 198): *A spacetime* $(\mathcal{M}, g_{\mu\nu})$ *is stably causal if and only if there exists a differentiable function* f *on* $\mathcal{M}$ *such that* $\nabla^\mu f$ *is a future directed timelike vector field.* Here $\mathcal{M}$ is a manifold and $g_{\mu\nu}$ is metric with Lorentzian signature. Note, that the notion of *stable causality* implies that the spacetime $(\mathcal{M},\, g_{\mu\nu})$ possesses no CCCs and thus no causal paradoxes can arise in this case. The theorem above has a kinematic origin and does not rely on the dynamical equations. In the case of the effective acoustic geometry the acoustic metric $G^{-1}_{\mu\nu}$ plays the role of $g_{\mu\nu}$ and the function f serves as the "global time function" of the emergent spacetime $\left(\mathcal{M},\, G^{-1}_{\mu\nu}\right)$. For example, in the Minkowski spacetime filled with the scalar field "ether" one can take the Minkowski time t of the rest frame, where this field is homogeneous, as the global time function. Then we have

$$G^{\mu\nu}\partial_\mu t \partial_\nu t = \frac{c_s}{\mathcal{L}_{,X}} g^{00}\left(1 + 2X\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right) = \frac{g^{00}}{\mathcal{L}_{,X} c_s}. \tag{3.1}$$

Even for those cases when the speed of perturbations can exceed the speed of light, $c_s > 1$, this expression is positive, provided that $\mathcal{L}_{,X} > 0$, and the hyperbolicity condition (2.5) is satisfied.

Thus $\partial_\mu t$ is timelike with respect to the effective metric $G^{-1}_{\mu\nu}$; hence the conditions of the theorem above are met and no CCCs can exist.

Now we consider the Minkowski spacetime with an arbitrary inhomogeneous background $\phi_0(x)$ and verify under which conditions one can find a global time t for both geometries $g_{\mu\nu}$ and $G^{-1}_{\mu\nu}$ and thus guarantee the absence of CCCs. Let us take the Minkowski t, $\eta^{\mu\nu}\partial_\mu t\partial_\nu t = 1$, and check whether this time can also be used as a global time for $G^{-1}_{\mu\nu}$. We have

$$G^{\mu\nu}\partial_\mu t\partial_\nu t = \frac{c_s}{\mathcal{L}_{,X}}\left[1+\left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)\left(\partial_\mu t\nabla^\mu\phi_0\right)^2\right] = \frac{c_s}{\mathcal{L}_{,X}}\left[1+\left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)\dot{\phi}_0^2\right], \tag{3.2}$$

and assuming that $c_s > 0$, $\mathcal{L}_{,X} > 0$ we arrive to the conclusion that t is a global time for emergent spacetime provided

$$1+\left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)\left(\dot{\phi}_0\left(x^\mu\right)\right)^2 > 0, \tag{3.3}$$

holds everywhere on the manifold $\mathcal{M}$. This inequality is obviously always satisfied in the subluminal case. It can be rewritten in the following form

$$1+c_s^2\left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)\left(\vec{\nabla}\phi_0\left(x^\mu\right)\right)^2 > 0, \tag{3.4}$$

from where it is obvious that, if the spatial derivatives are sufficiently small then this condition can also be satisfied even if $c_s > 1$. Note that the breaking of the above condition for some background field configuration $\phi_0(x)$ does not automatically mean the appearance of the CCCs. This just tells us that the time coordinate t cannot be used as the global time coordinate. However it does not exclude the possibility that there exists another function serving as the global time. Only, if one can prove that such global time for both metrics does not exist at all, then there arise causal paradoxes.

In the case of the Friedmann universe with "superluminal" scalar field, one can choose the cosmological time t as the global time function and then we again arrive to (3.1), thus concluding that there exist no CCCs. In particular, the k-*essence* models, where the superluminal propagation is the generic property of the fluctuations during some stage of expansion of the universe [2, 32], *do not lead to causal paradoxes* contrary to the claim by [2, 30].

The absence of the closed causal curves in the Friedmann universe with k-*essence* can also be seen directly by calculating of the "effective" line element (2.13). Taking into account that the Friedmann metric is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)d\mathbf{x}^2, \tag{3.5}$$

we find that the line element (2.13), corresponding to the effective acoustic metric, is

$$dS^2 = G^{-1}_{\mu\nu}dx^\mu dx^\nu = \frac{\mathcal{L}_{,X}}{c_s}\left(c_s^2dt^2 - a^2(t)d\mathbf{x}^2\right). \tag{3.6}$$

The theory under consideration is generally covariant. After making redefinitions, $\sqrt{\mathcal{L}_{,X}c_s}dt \to dt$, and, $a^2(t)\mathcal{L}_{,X}/c_s \to a^2(t)$, the line element (3.6) reduces to the interval for the Friedmann universe (3.5), where obviously no causality violation can occur. Thus we conclude that both the k-*essence* [26] and the "superluminal" inflation with large gravity waves [27] are completely safe and legitimate on the side of causality.

When $X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0$ is positive everywhere in the spacetime the background field itself can be used as the global time function. Indeed for general gravitational background $g_{\mu\nu}$ and $c_s > 0$, $\mathcal{L}_{,X} > 0$ we have

$$g^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0 > 0 \text{ and } G^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0 = \frac{2X}{\mathcal{L}_{,X}c_s} > 0,$$

and due to the fact that $X > 0$ the sign in front $\nabla^\mu \phi_0$ can be chosen so that the vector $\nabla^\mu \phi_0$ is always future directed on $\mathcal{M}$. Therefore $\phi_0(x)$ or $(-\phi_0(x)$ if necessary) can serve as a global time in both spacetimes $(\mathcal{M},\, g_{\mu\nu})$ and $\left(\mathcal{M},\, G^{-1}_{\mu\nu}\right)$, and no causal paradoxes arise.

In particular this is applicable for the accretion of the "superluminal" scalar field onto the Schwarzschild black hole [29]. In this case sound horizon is located inside the Schwarzschild radius and therefore the Schwarzschild time coordinate cannot be used as a global time function. However, $X > 0$ outside the acoustic horizon (see [29]) and in accordance with the theorem and discussion above we can take ϕ as the global time coordinate and hence the acoustic spacetime is stably causal.

In all examples above we have considered the "superluminal" acoustic metric. Thus, if there exist no CCCs in $\left(\mathcal{M},\, G^{-1}_{\mu\nu}\right)$ then there are no CCCs with respect to metric $g_{\mu\nu}$ because acoustic cone is larger than the light cone. It may happen that in some cases it is not enough to prove that there no CCCs separately in $\left(\mathcal{M},\, G^{-1}_{\mu\nu}\right)$ and $(\mathcal{M},\, g_{\mu\nu})$ and one has to use the maximal cone or introduce an artificial cone [31] encompassing all cones arising in the problem. It is interesting to note that, if the k-*essence* realizes both "superluminal" and subluminal speed of sound in the different regions of the manifold, then there exist hypersurface where the k-*essence* metric is conformally equivalent to the $g_{\mu\nu}$ and one can smoothly glue the maximal cones together everywhere on $\mathcal{M}$. After that one can consider a new "artificial metric" $G^{\Sigma}_{\mu\nu}$ as determining the complete causal structure of the manifold.

We would like to point out that although the theorem on stable causality allowed us to prove that there is no causal paradoxes in those cases we considered above, it is no guaranteed that CCCs cannot arise for some other backgrounds. Indeed, in [8] the authors have found some configurations of fields possessing CCCs: one for the scalar field with non-canonical kinetic term and another for the "wrong"-signed Euler-Heisenberg system. In both cases the small perturbations propagate superluminally on rather non-trivial backgrounds. We will pursue this issue further in Section 6.

4. Which initial data are allowed for the well posed Cauchy problem?

Using the theorem on stable causality we have proven that the "superluminal" k-*essence* does not lead to any causal paradoxes for cosmological solutions and for accretion onto black hole. However, the consideration above is of a kinematic nature and it does not deal with the question how to pose the Cauchy problem for the background field ϕ_0 and it's perturbations π.

It was pointed out in [8] that in the reference frame of the spacecraft moving with respect to nontrivial background, where $c_s > 1$, with the speed $v = 1/c_s$ the Cauchy problem for small perturbations π is ill posed. This happens because the hypersurface of the constant proper time t' of the astronaut is a null-like with respect to the acoustic metric $G^{-1}_{\mu\nu}$. Hence $t' = const$ is tangential to the characteristic surface (or sonic cone see Appendix A) and cannot be used to formulate the Cauchy problem for perturbations which "live" in this acoustic metric. Intuitively this happens because the perturbations propagate instantaneously with respect to the hypersurface $t' = const$. Moreover, for $v > 1/c_s$, the sonic cone deeps below the surface $t' = const$ (see Figs. 3 and 2) and in the spacetimes of dimension $D > 2$ the Cauchy problem is ill posed as well because there always exist two directions along which the perturbations propagate "instantaneously" in time t' (red vectors in Fig. 3). This tell us that not every imaginable configuration of the background can be realized as the result of evolution of the system with the well formulated Cauchy problem and hence not every set of initial conditions for the scalar field is allowed.

In this Section we will find under which restrictions on the initial configuration of the scalar field the Cauchy problem for equation (2.3) is well-posed. For this purpose it is more convenient not to split the scalar field into background and perturbations and consider instead the total value of the field $\phi = \phi_0 + \pi$. The k-*essence* field interacts with gravity and therefore for consistency one has to consider the coupled system of equations for the gravitational metric $g_{\mu\nu}$ and the k-*essence* field

ϕ. In this case the Cauchy problem is well posed only if the initial data are set up on a hypersurface Σ which is simultaneously spacelike in both metrics: $g_{\mu\nu}$ and $G^{-1}_{\mu\nu}$ (for details see P. 251 of Ref. [39] and Refs. [40], [36], [41]). We will work in the synchronous coordinate system, where the metric takes the form

$$ds^2 = dt^2 - \gamma_{ik} dx^i dx^k, \tag{4.1}$$

and select the spacelike in $g_{\mu\nu}$ hypersurface Σ to be a constant time hypersurface $t = t_0$. The 1-form $\partial_\mu t$ vanishes on any vector R^μ tangential to Σ: $R^\mu \partial_\mu t = 0$ (see Fig. 3). This 1-form is timelike with respect to the gravitational metric $g_{\mu\nu}$, that is $g^{\mu\nu}\partial_\mu t \partial_\nu t > 0$. In case when Lagrangian for k-*essence* depends at maximum on the first derivatives of scalar field the initial conditions which completely specify the unambiguous solution of the equations of motion are the initial field configuration $\phi(\mathbf{x})$ and and it's first time derivative $\dot\phi(\mathbf{x}) \equiv (g^{\mu\nu}\partial_\mu t \partial_\nu \phi)_\Sigma$. Given these initial conditions one can calculate the metric $G^{-1}_{\mu\nu}$ and consequently the influence cone at every point on Σ. First we have to require that for a given set of initial data the hyperbolicity condition (2.5) is not violated. This imposes the following restriction on the allowed initial values $\phi(\mathbf{x})$ and $\dot\phi(\mathbf{x})$:

$$c_s^{-2} = 1 + \left[\left(\dot\phi(\mathbf{x})\right)^2 - \left(\vec\nabla\phi(\mathbf{x})\right)^2\right] \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} > 0, \tag{4.2}$$

where we have denoted $\left(\vec\nabla\phi(\mathbf{x})\right)^2 = \gamma^{ik}\partial_i\phi\partial_k\phi$. In addition we have to require that the hypersurface Σ is spacelike also with respect to emergent metric $G^{\mu\nu}$, that is, for every vector R^μ, tangential to Σ, we have $G^{-1}_{\mu\nu}R^\mu R^\nu < 0$, or

$$1 + c_s^2 \left(\vec\nabla\phi(\mathbf{x})\right)^2 \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} > 0. \tag{4.3}$$

If at some point on Σ the vector R^μ becomes null-like with respect to $G^{-1}_{\mu\nu}$, that is, $G^{-1}_{\mu\nu}R^\mu R^\nu = 0$, the signals propagate instantaneously (red propagation vectors from cone B on Fig. 3) and one cannot guarantee the continuous dependence on the initial data or even the existence and uniqueness of the solution, see e.g. [52]. Using (2.6) the last inequality can be rewritten as

$$c_s^2 \left(1 + \left(\dot\phi(\mathbf{x})\right)^2 \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right) > 0. \tag{4.4}$$

Therefore, given Lagrangian $\mathcal{L}(\phi, X)$ and hypersurface Σ one has to restrict the initial data $\left(\phi(\mathbf{x}), \dot\phi(\mathbf{x})\right)$ by inequalities (4.2) and (4.3) (or equivalently (4.4)), to have a well posed Cauchy problem. The condition (4.4) is always satisfied in the subluminal case for which $\mathcal{L}_{,XX}/\mathcal{L}_{,X} \geq 0$. In addition, we conclude that, if these conditions are satisfied everywhere on the manifold $\mathcal{M}$ and the selected synchronous frame is nonsingular in $\mathcal{M}$, then time t plays the role of global time and in accordance with the theorem about stable causality no causal paradoxes arise in this case.

As a concrete application of the conditions derived let us find which restrictions should satisfy the admissible initial conditions for the low energy effective field theory with Lagrangian $\mathcal{L}(X) \simeq X - X^2/\mu^4 + ...$, where μ is a cut off scale Ref. [8]. In this case (4.4) imply that not only $X \ll \mu^4$, but also $\left(\dot\phi(\mathbf{x})\right)^2 \ll \mu^4$ and $\left(\vec\nabla\phi(\mathbf{x})\right)^2 \ll \mu^4$. Note that these restrictions can be rewritten in the Lorentz invariant way: for example the first condition takes the form $(g^{\mu\nu}\partial_\mu t \partial_\nu \phi)^2 \ll \mu^4$.

Finally let us note that even well-posed Cauchy problem cannot guarantee the global existence of the unique solution for nonlinear system of the equations of motion: for example, the solution can develop caustics [58] or can become multi-valued [61].

5. How to pose the initial conditions in a fast moving spacecraft?

In this section we resolve "paradoxes" which at first glance seems arising in the case of superluminal propagation of perturbations [8, 30, 2] when one tries to formulate the Cauchy problem in a fast

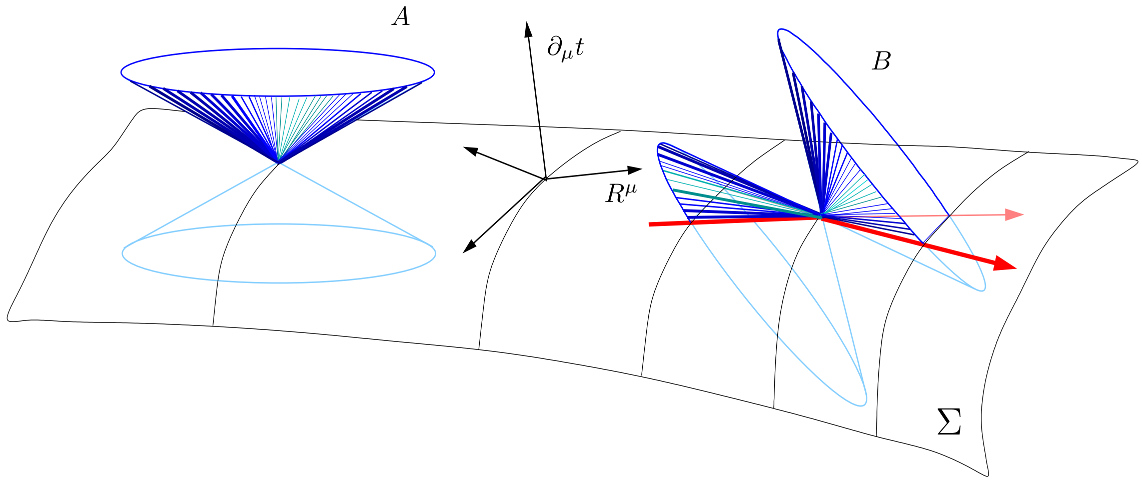

Figure 3: The Cauchy problem for the equation of motion of k-*essence* is set up on the hypersurface Σ: $t = t_0$. The vector R^μ is tangential to Σ: $R^\mu \partial_\mu t = 0$. For the hyperbolic equation of motion, the Cauchy problem is well posed provided that $\partial_\mu t$ is timelike with respect to $G^{\mu\nu}$ everywhere on Σ, or, equivalently, the hypersurface Σ is spacelike with respect to $G^{\mu\nu}$ (the cone A in the figure). The cone B represents an ill-posed Cauchy problem for the hyperbolic equation. In particular the red propagation vectors are tangent to Σ.

moving spacecraft. To simplify the consideration we restrict ourselves by purely kinetic k-*essence*, for which $\mathcal{L}(\phi, X) = \mathcal{L}(X)$ and assume that for the background solution $X_0 = const > 0$ and $c_s > 1$. This is a reasonable approximation for more general backgrounds with $X_0 > 0$ on the scales much smaller than the curvature scale of the emergent geometry $G^{-1}_{\mu\nu}$. There is always the preferred reference frame $\left(t, x^i\right)$ in which the background is isotropic and homogeneous. We refer to this frame as the *rest frame*. In the presence of an external source δJ equation (2.7) in this frame takes the following form

$$\partial_t^2 \pi - c_s^2 \triangle_x \pi = \xi \delta J, \tag{5.1}$$

where $\xi \equiv \left(c_s^2/\mathcal{L}_{,X}\right)$, for details see Appendix B, equations (B.20) and (B.1). Now let us consider a spacecraft moving in x-direction with velocity v through the k-*essence* background and denote the Lorentz boosted comoving spacecraft coordinates by $\left(t', x'^i\right)$. As we have already mentioned above, if the velocity of the spacecraft is larger than c^2/c_s then the Cauchy problem for π cannot be well posed on the hypersurface $t' = const$ [3]. After Lorentz transformation to comoving spacecraft frame, equation (5.1) becomes

$$\left(1 - \frac{v^2}{c^2}\right)^{-1} \left[\left(1 - \frac{c_s^2 v^2}{c^4}\right) \partial_{t'}^2 \pi - 2v \left(1 - \frac{c_s^2}{c^2}\right) \partial_{t'} \partial_{x'} \pi + \left(v^2 - c_s^2\right) \partial_{x'}^2 \pi\right] - c_s^2 \partial_J \partial_J \pi = \xi \delta J, \tag{5.2}$$

where prime denotes comoving coordinates and index $J = 2, 3, ...$ stands for the spatial directions other than x' [note that in Ref. [8] the factor $\left(1 - v^2/c^2\right)^{-1}$ in front of squire brackets is missing]. For $v = c^2/c_s$ the second time derivative drops out of (5.2) and the necessary conditions for applicability of the Cauchy-Kowalewski theorem are not satisfied; hence the existence and uniqueness of the solution (5.2) are not guaranteed. For $v > c^2/c_s$ the necessary conditions of the Cauchy-Kowalewski theorem are met and the unique solution of (5.2) exists; however, this solution contains exponentially growing modes in the spatial directions, perpendicular to x'. Indeed, substituting

$$\pi \propto \exp\left(-i\omega' t' + ik_{x'} x' + ik_J x^J\right),$$

[3] Throughout this section we explicitly write the speed of light c and without loss of generality we assume $v > 0$.

in (5.2) we find that in the boosted frame:

$$\omega'_{\pm} = \left(1 - \frac{v^2 c_s^2}{c^4}\right)^{-1} \left\{ k_{x'} v \left(\frac{c_s^2}{c^2} - 1\right) \pm c_s \sqrt{\left(1 - \frac{v^2}{c^2}\right) \left[k_{x'}^2 \left(1 - \frac{v^2}{c^2}\right) - \left(\frac{v^2 c_s^2}{c^4} - 1\right) k_\perp^2 \right]} \right\}. \quad (5.3)$$

where we have denoted $k_\perp = \{k_J\}$ and $k_\perp^2 = k_J k_J$. For $D = 2$, when $k_\perp = 0$, the frequencies ω are always real and no instability modes exist (note that $v < c$). However, if $D > 2$ and $v > c^2/c_s$ then for

$$k_\perp^2 > k_{x'}^2 \left(\frac{1 - v^2/c^2}{v^2 c_s^2/c^4 - 1}\right), \quad (5.4)$$

the general solution of (5.2) contains exponentially growing modes. Note that these are the high frequency modes and hence the instability would imply catastrophic consequences for the theory. At first glance, this looks like a paradox, because equation (5.1), which has no unstable solutions in the rest frame, acquired exponentially unstable solutions in the boosted frame. On the other hand, any solution of (5.1) after performing the Lorentz transformation with $v > c^2/c_s$ does not contain exponentially growing modes with $k_\perp^2$ satisfying (5.4). Indeed, given $(k_x, k_\perp)$ in the rest frame one can perform the Lorentz transformation and obtain:

$$\{\omega',\, k_{x'},\, k'_\perp\} = \left\{ \frac{\omega + v k_x}{\sqrt{1 - v^2/c^2}},\, \frac{k_x + \omega v/c^2}{\sqrt{1 - v^2/c^2}},\, k_\perp \right\}, \quad (5.5)$$

were $\omega = \pm c_s \sqrt{k_x^2 + k_\perp^2}$. Expressing ω' via $k_{x'}$ and $k_\perp$ we again arrive to (5.3). However, it follows from (5.5) that if $v > c^2/c_s$ then the components of the Lorentz boosted wavevector satisfy the condition

$$k_\perp^2 \leq k_{x'}^2 \left(\frac{1 - v^2/c^2}{v^2 c_s^2/c^4 - 1}\right), \quad (5.6)$$

and hence unstable modes are not present. This raises the question whether the unstable modes which cannot be generated in the rest frame of k-*essence*, can nevertheless be exited by any physical device in the spacecraft. We will show below that such device does not exist. With this purpose we have to find first the Greens function in both frames.

Let us begin with two-dimensional spacetime. In this case the retarded Green's function for (5.1) in the rest frame (rf) is (see e.g. [52]):

$$G_R^{\mathrm{rf}}(t, x) = \frac{1}{2c_s} \theta\left(c_s t - |x|\right). \quad (5.7)$$

In the boosted Lorentz frame it becomes

$$G_R^{\mathrm{rf}}(t', x') = \frac{1}{2c_s} \theta\left(\frac{c_s\left(t' + v x'/c^2\right) - |x' + v t'|}{\sqrt{1 - v^2/c^2}} \right). \quad (5.8)$$

For $c_s v < c^2$, the Fourier transform of (5.8) is the retarded in t' Green's function:

$$G_R^{\mathrm{rf}}(t', k') = \frac{\theta(t')}{2 i c_s k'} \left(e^{i\omega'_+ t'} - e^{i\omega'_- t'} \right), \quad (5.9)$$

whereas for $c_s v > c^2$ it is given by:

$$G_R^{\mathrm{rf}}(t', k') = -\frac{\theta(t') e^{i\omega'_+ t'} + \theta(-t') e^{i\omega'_- t'}}{2 i c_s k'}. \quad (5.10)$$

This Green's function corresponds to the Feynman's boundary conditions in the boosted frame. Thus, in the fast moving spacecraft, the *retarded* Green's function (5.10), obtained as a result of

Lorentz transformation from (5.7) looks like a mixture of the retarded [proportional to $\theta(t')$] and the advanced [proportional to $\theta(-t')$] Green's functions with respect to the spacecraft time t'. In fact, the situation is even more complicated. If from the very beginning we work in the comoving spacecraft frame (sc), then solving (5.2) we obtain the following expression for the retarded Green's function,

$$G_R^{\rm sc}(t',k') = \frac{\theta(t')}{2ik'c_s}\left(e^{i\omega'_+t'} - e^{i\omega'_-t'}\right). \tag{5.11}$$

which coincides with equation (5.7), only if $c_s v < c^2$. However, for fast moving spacecraft, $c_s v > c^2$, formula (5.11) does not coincide with formula (5.10).

The situation is more interesting in the four dimensional spacetime. Similar to the 2d case, after we apply the Lorentz boost to the retarded (in the rest frame) Green's function (see e.g. [52])

$$G_R^{\rm rf}(t,x^i) = \frac{\theta\left(t\right)}{2c_s\pi}\delta\left(c_s^2t^2 - |x|^2\right), \tag{5.12}$$

and calculate its Fourier transform (see Appendix E for the details) we find that for the slowly moving spacecraft, $vc_s < c^2$,

$$G_R^{\rm rf}(t',k') = \frac{\theta\left(t'\right)}{2ic_s}\left(k_{x'}^2 + k_\perp^2\frac{1-c_s^2v^2/c^4}{1-v^2/c^2}\right)^{-1/2}\left({\rm e}^{i\omega'_+t'} - {\rm e}^{i\omega'_-t'}\right). \tag{5.13}$$

That is, the resulting Green's function is also retarded with respect to the spacecraft time t'. On the other hand, for the fast moving spacecraft, $vc_s > c^2$, we obtain:

$$G_R^{\rm rf}(t',k') = -\frac{1}{2ic_s}\left(k_{x'}^2 + k_\perp^2\frac{1-c_s^2v^2/c^4}{1-v^2/c^2}\right)^{-1/2}\left(\theta\left(t'\right){\rm e}^{i\omega'_+t'} + \theta\left(-t'\right){\rm e}^{i\omega'_-t'}\right). \tag{5.14}$$

Similar to the 2d case formula (5.14) is the Feynman Green's function in the spacecraft frame. Note that formula (5.14) can be rewritten as:

$$\begin{aligned} G_R^{\rm rf}(t',k') &= \frac{1}{2c_s}\left(k_{x'}^2 + k_\perp^2\frac{1-c_s^2v^2/c^4}{1-v^2/c^2}\right)^{-1/2} \times \\ &\times \exp\left(-ik_{x'}vt'\frac{1-c_s^2/c^2}{1-c_s^2v^2/c^4} - \frac{1-v^2/c^2}{c_s^2v^2/c^4-1}c_s\left|t'\right|\sqrt{k_\perp^2\frac{1-v^2/c^2}{c_s^2v^2/c^4-1} - k_{x'}^2}\right). \end{aligned} \tag{5.15}$$

It is obvious from here that the modes with large $k_\perp$ are exponentially suppressed and therefore very high frequency source δJ cannot excite perturbations with $k_\perp^2$ satisfying inequality (5.4).

In the spacecraft frame the retarded Green's function calculated directly for Fourier modes of (5.2) is:

$$G_R^{\rm sc}(t',k') = \frac{\theta\left(t'\right)}{2ic_s}\left(k_{x'}^2 + k_\perp^2\frac{1-c_s^2v^2/c^4}{1-v^2/c^2}\right)^{-1/2}\left({\rm e}^{i\omega'_+t'} - {\rm e}^{i\omega'_-t'}\right).$$

It coincides with Green's function (5.13), obtained by applying the Lorentz transformation, only in the case of slow motion with $v < c^2/c_s$. However, the results drastically differ for the fast moving spacecraft - compare equations (5.13) and (5.14). The function $G_R^{\rm sc}(t',k')$ contains exponentially growing modes for sufficiently large $k_\perp$ and it's Fourier transform to coordinate space $G_R^{\rm sc}(t',x')$ does not exist. Physically this means that we have failed to find the Green's function, which describes the propagation of the signal which the source δJ in the fast moving spacecraft tries to send in the direction of growing t'. Instead, the response to any source in the spacecraft is always driven by (5.15) (or the Lorentz transformed Green's function in the rest frame (5.12)). Because we cannot send a signal in the direction of growing t' one cannot associate growing t' with the arrow of time contrary to the claims in [30].

Now we will discuss in more details how the problem of initial conditions for perturbations π must be correctly formulated in the fast moving spacecraft. The first question here whether the fast moving astronaut can create an arbitrary initial field configurations π and $\dot{\pi}$ at a given moment of his proper time $t_1' = const$. This hypersurface is not *space-like* with respect to the metric $G^{-1}_{\mu\nu}$ and therefore as it follows from the consideration in the previous section the Cauchy problem is not well posed on it. Hence not all possible configurations are admissible on this hypersurface but only those which could be obtained as a result of evolution of some initial configuration chosen on the hypersurface which is simultaneously spacelike with respect to both metrics $g_{\mu\nu}$ and $G^{-1}_{\mu\nu}$. If the astronaut disturbs the background with some device (source function δJ) which he/she switches off at the moment of time t_1', then the resulting configuration of the field on the hypersurface $t_1' = const$ obtained using the correct Green's function (5.15) will always satisfy the conditions needed for unambiguous prediction of the field configuration everywhere in the spacetime irrespective of the source $\delta J(x)$. The presence of the advanced mode in this Green's function plays an important role in obtaining a consistent field configuration on $t_1' = const$. Thus we see that not "everything" is in the hand of the astronaut: he has no "complete freedom" in the choice of the "initial" field configuration at time t_1'. Nonrecognition of this fact leads to the fictitious causal paradoxes discussed in the literature [2, 30].

For a slowly moving spacecraft, $v < c^2/c_s$, the retarded Green's function in the rest frame is transformed in the retarded Green's function in the spacecraft frame. Therefore we can obtain any a priori given field configuration on the hypersurface $t_1' = const$ by arranging the source function δJ in the corresponding way. Thus, the choice of the initial conditions for the perturbations at $t_1' = const$ is entirely in the hand of the astronaut. This is in complete agreement with our previous consideration because in the slowly moving spacecraft the hypersurface $t_1' = const$ is spacelike with respect to both metrics.

The appearance of the advance part in the correct Green's function for the fast moving spacecraft still looks a little bit strange because according to the clocks of the astronaut the head of the spacecraft can "feel" signals sent at the same moment of time by a device installed on the stern of the spacecraft. However, in this case the proper time of the astronaut is simply not a good coordinate for the time ordering of the events at different points of the space related by the k-*essence* superluminal signals. The causality is also preserved in this case but it is determined by the superluminal k-*essence* cone which is larger than the light cone and as we have already seen no causal paradoxes arise in this case. If the astronaut synchronizes his clocks using the superluminal sonic signals then the new time coordinate $\tilde{t}$ becomes a good coordinate for the time ordering of the causal events in different points of the space. The hypersurface $\tilde{t} = const$ being spacelike in both metrics can then be used as the initial hypersurface for the well posed Cauchy problem in the fast moving spacecraft, that is, any initial configuration of the field can be freely created by the astronaut on this hypersurface. In the "well synchronized' reference frame $\left(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}\right)$ the equation of motion for perturbations (5.1) takes the same form as in the rest frame of the k-*essence* background:

$$\partial^2_{\tilde{t}}\pi - c_s^2 \triangle_{\tilde{x}} \pi = \xi\delta J. \tag{5.16}$$

It follows from here that

$$\omega_{\pm} = \pm c_s \sqrt{k^2_{\tilde{x}} + k^2_{\tilde{y}} + k^2_{\tilde{z}}},$$

and hence no exponentially growing modes exist for any $k_{\tilde{x}}$, $k_{\tilde{y}}$ and $k_{\tilde{z}}$.

The causal Green's function in the spacecraft frame contains only the retarded with respect to the time $\tilde{t}$ part. For example, in four-dimensional spacetime it is given by

$$G_R^{\mathrm{sc}}(\tilde{t}, \tilde{x}^i) = \frac{\theta\left(\tilde{t}\right)}{2c_s\pi}\delta\left(c_s^2\tilde{t}^2 - |\tilde{x}^i|^2\right). \tag{5.17}$$

This result can be obtained either by applying the Lorentz transformation with the invariant speed c_s to (5.7), or directly by solving equation (5.16). Thus, no paradoxes with Green's functions arise for the superluminal perturbations. The same conclusions are valid in 4d spacetime.

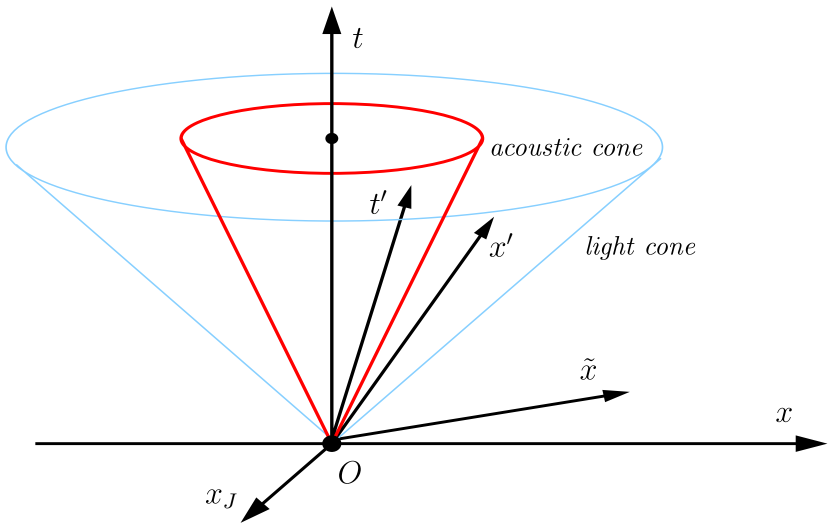

Figure 4: It is shown how one can create a would -be "paradox" similar to that discussed in this section, without involving any superluminal signals. The fluid is at rest and the perturbations propagate subluminally in the fluid, $c_s < c$. The reference frame $(t',\, x')$ is connected to the rest frame by the Lorentz boost with the invariant speed c_s. If the boost speed v is such that $c_s/c < v/c_s < 1$, then the hypersurface of constant t' is inside the light cone and the Cauchy problem for the electromagnetic field is ill posed in this reference frame. Instead, one should use the "correct" frame $(\tilde{t},\, \tilde{x})$, obtained by the Lorentz boost with the invariant fastest speed $c = 1$. In this frame the Cauchy problem is well-posed.

To make the consideration above even more transparent we conclude this section by considering analogous situation with *no* superluminal signals involved. Namely, we take a fluid at rest with a *subluminal* speed of sound, $c_s < c$. Then we can make the Lorentz transformation using the invariant speed c_s:

$$t' = \frac{t - vx/c_s^2}{\sqrt{1 - v^2/c_s^2}}, \;\; x' = \frac{x - vt}{\sqrt{1 - v^2/c_s^2}}, \;\; x'_J = x_J.$$

If the speed v is such that $c_s/c < v/c_s < 1$, then the hypersurface of constant t' is *inside* the light cone (see Fig. 4) and it is obvious that one cannot formulate the Cauchy problem for the electromagnetic field on the hypersurface $t' = const$. Instead, the Cauchy problem for the electromagnetic field can be well posed on the hypersurface $\tilde{t} = const$ defined by the "correct" Lorentz transformation, with the invariant speed c:

$$\tilde{t} = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}, \;\; \tilde{x} = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \;\; \tilde{x}_J = x_J,$$

(see Fig. 4). This consideration is fully equivalent to those one above with the only replacement $c_s \leftrightarrow c$.

Thus we have shown that no physical paradoxes arise in the case when we have superluminal propagation of small perturbations on the background.

6. Chronology protection

It was claimed in [8] that the theories with superluminal propagation are plagued by closed causal

curves (CCC). We will argue here that the superluminal propagation cannot be the sole reason for the appearance of CCC and moreover this problem can be avoided in this case in the same way as in General Relativity.

It is well know that General Relativity admits the spacetimes with the closed causal curves without involving any superluminal fields into consideration. Among examples of such spacetimes are: Gdel's cosmological model [21], Stockum's rotating dust cylinder [59], wormholes [25], Gott's solution for two infinitely long strings [22] and others [24]. A prominent time-machine model was suggested recently by Ori [23]. In this model, made solely of vacuum and dust, the spacetime evolves from a regular normal asymptotically flat state without CCCs and only later on develops CCCs without violating the weak, dominant and strong energy conditions. Thus, we see that initially "good" spacetime might in principle evolve to a state where the chronology is violated and the General Relativity does not by itself explains these strange phenomena. Therefore one needs to invoke some additional principle(s) to avoid the pathological situations with CCCs. With this purpose Hawking suggested the *Chronology Protection Conjecture*, which states that the laws of physics must prohibit the appearance of the closed timelike curves [20]. In [20] it was argued that in the situation when the timelike curve is ready to close, the vacuum polarization effects become very large and the backreaction of quantum fields prevents the appearance of closed timelike curves.

Similarly to General Relativity, one might assume that in the case of superluminal propagation the chronology protection conjecture is valid as well. For example, the chronology protection was already invoked to exclude the causality violation in the case of two pairs of Casimir plates [19], in which photons propagate faster than light due to the Scharnhorst effect [16].

Once we employ the chronology protection principle, no constructions admitting CCCs, similar to those presented in [8], may become possible.

In fact, the first example in [8] with two finite fast moving bubbles made of superluminal scalar field (see Fig. 2 in Ref. [8]) is quite similar to the "time machine" involving two pair of Casimir plates. In the latter case the chronology protection excludes the existence of CCCs. Here the situation is a little bit more involved. In the example with the bubbles the background is not a free solution of the equation of motion (2.3). Indeed as it was pointed out in [8] the fast moving bubbles have to be separated in the direction orthogonal to the direction of motion. On the other hand they have to be connected by light. However, if this were a free solution, then the bubbles would expand with the speed of light and collide at the same moment of time, or even before the closed causal curve would be formed. Thus an external source $J(x)$ of the scalar field is required in order to produce this acausal background. However, without clear idea about the origin of this source and possible backreaction effects the physical interpretation of this "time machine" is obscure. It is well known that admitting all possible sources of gravitational field one can obtain almost any possible even acausal solutions in general relativity. Finally, generalizing the Hawking conjecture to the case of scalar field one can argue that the backreaction of quantum fluctuations of perturbations π around ϕ become large before CCC is formed thus destroying the classical solution imposed by the external source $J(x)$ and preventing the formation of CCC.

The other example considered in [8] involves non-linear electrodynamics. The electromagnetic field is created by charge currents, serving as a source. Thus, unlike the previous example, one can control the strength of the field, simply changing the configuration of charges. The electromagnetic part of the Lagrangian is the "wrong"-signed Euler-Heisenberg Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \alpha\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + ..., \tag{6.1}$$

with a small positive α. For such a system the propagation of light in a non-trivial background is superluminal. As a consequence, a cylindrical capacitor with the current-carrying solenoid leads to the appearance of the CCCs, provided the electrical and magnetic fields inside the capacitor are

large enough (see Fig. 3 in [8]). In this case one may invoke a simplified version of the chronology protection conjecture. In fact, let us begin with some "good" initial conditions in the capacitor, namely, with electric and magnetic fields being not too large, so that no CCCs exist. Then we increase the current in the solenoid and the voltage between the plates of a capacitor in order to increase the strength of the fields. When the causal curves become almost closed, the expectation value of the energy-momentum tensor for the "quasi-photons" on this classical electromagnetic background becomes very large due to the quantum vacuum polarization effects. In the limit when the causal cone becomes horizontal, the energy density of the field in the capacitor tends to infinity and the capacitor will be broken before the CCCs will be formed.

Thus we conclude that concerning the causal paradoxes the situation in the theories with superluminal propagation on the non-trivial backgrounds is not much worse than in General Relativity. In fact, in this respect the similarity between these two theories goes even much deeper than it looks at the first glance. For example, let us imagine a time machine which is constructed with the help of superluminal propagation in non-trivial background produced by the external source $J(x)$, e.g., similar to those described in [8]. Then we can identify the *effective metric* $G^{\mu\nu}$ for this system with the *gravitational metric* $g^{\mu\nu}$ of some spacetime produced by an energy-momentum tensor $T^{(J)}_{\mu\nu}(x)$. Put differently, once having the effective metric, we can find spacetime where the gravitational metric is $G^{\mu\nu}$. In this spacetime the time machine exists as well. Remarkably, now the gravitation (or light) signals are used to make CCCs. The spacetime with the metric $G^{\mu\nu}$ is the solution of Einstein equations with the energy-momentum tensor $T^{(J)}_{\mu\nu}$ calculated substituting the metric in the Einstein equations. After that one could try to find such theories and such fields configurations on which their resulting energy momentum tensor is equal to $T^{(J)}_{\mu\nu}(x)$ consistently with equations of motion. One can, in principle, argue, that in the case when the CCCs exist the energy-momentum tensor mights have some undesired properties, for example, it would violate the Week Energy Condition (WEC). However, in several known examples with CCCs the WEC is satisfied, see, e.g. Refs. [21, 22, 59]. Moreover, the system found by A. Ori [23] possesses CCCs and satisfies the week, dominant and strong energy conditions. Thus the violation of the energy conditions is not an inherent property of the spacetimes with CCCs. Therefore the question, whether the spacetime constructed by the procedure described above requires "bad" energy-momentum tensor or not, must be studied separately in each particular case.

Moreover the correspondence $G^{\mu\nu} \leftrightarrow g^{\mu\nu}$, $J \leftrightarrow T^{(J)}_{\mu\nu}$ can also be used to learn more about *Chronology Protection Conjecture* and time-machines in General Relativity with the help of more simple theory. It is well-known that Analogue Gravity [60] gives more simple and intuitively clear way to investigate the properties of Hawking radiation, the effects of Lorentz symmetry breaking, transplanckian problem *etc.,* by using the small perturbations in the fluids instead of direct implication of General Relativity. In a similar way, *analogue time-machine* or analogue *Chronology Protection Conjecture* may provide one with a tool to check *Chronology Projection Conjecture* and the possibility of construction of time machines in General Relativity.

7. Is the gravitational metric universal?

It was argued in [3] that the same causal limits apply to all fields independent of the matter present, thus endowing the gravitational metric with the universal role. However, in the theories under consideration the causal limit is governed not by the gravitational metric $g_{\mu\nu}$, but by the effective acoustic metric $G^{-1}_{\mu\nu}$ and hence the gravity loses its universal role in this sense. Nevertheless, here we argue, that even in the case of the spontaneously broken Lorentz invariance with a superluminal propagation the gravitational metric $g_{\mu\nu}$ still keeps its universal role in the following sense. First in accordance with the discussion in 4 we remind that the Cauchy hypersurface for the field ϕ should anyway be a spacelike one in the gravitational metric. Thus in order to produce a background which

breaks the Lorentz invariance one has to respect the usual causality governed by the gravitational metric $g_{\mu\nu}$. Moreover, if a clump of the scalar field is created in a finite region surrounded by a trivial background, then the boundaries of the clump will generically propagate with the speed of light.

Indeed, let us consider a finite lump of non-trivial field configuration with smooth boundaries (see Fig. 5) and assume that the initial data $\left(\phi(\mathbf{x}), \dot{\phi}(\mathbf{x})\right)$ are specified in some finite spatial region R. These initial data are smooth everywhere (see Fig. 5) and satisfy the conditions (4.4) and (4.2), in particular the first derivatives of the field ϕ are continuous everywhere including the boundaries of the clump. If the system described by action (2.1) has at least one trivial solution $\phi = \phi_{\rm triv} = const$ with non-pathological acoustic geometry, then, as it follows from (2.3) and (2.12), the Lagrangian $\mathcal{L}(\phi, X)$ is at least twice differentiable at $(\phi, X) = (\phi_{\rm triv}, 0)$ and moreover $\mathcal{L}_{,X}(\phi_{\rm triv}, 0) \neq 0$. Thus for the theories of this type we have

$$\mathcal{L}(\phi, X) \simeq V(\phi) + K_1(\phi)X + K_2(\phi)X^2 + ... \tag{7.1}$$

in the vicinity of the trivial solution $\phi_{\rm triv}$ [4]. And as expected we conclude that the speed of sound for the small perturbations is equal to the speed of light in the vicinity of $\phi_{\rm triv}$ because any trivial solution and in particular a possible vacuum solution $\phi = 0$ does not violate the Lorentz invariance. Moreover, close to the boundaries of the clamp the initial data $\left(\phi(\mathbf{x}), \dot{\phi}(\mathbf{x})\right)$ can be considered as small perturbation around the trivial background and therefore the front of the clump propagates exactly with the speed of light in the vacuum. Thus, without preexisting nontrivial configuration of the scalar field the maximum speed of propagation never exceeds the speed of light and the causality is entirely determined by the usual gravitational metric only.

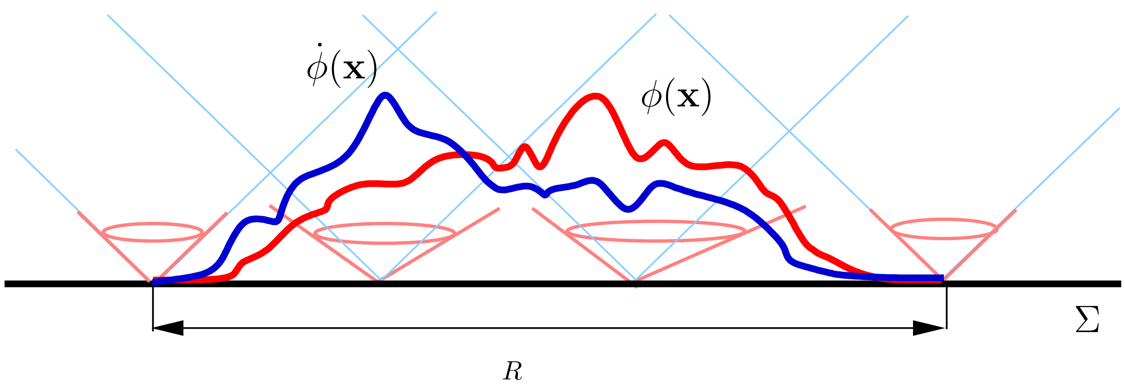

Figure 5: The figure shows that the gravitational metric $g_{\mu\nu}$ keeps its universal meaning even if the small perturbations on the non-trivial backgrounds propagate superluminally. If in the initial moment of time the non-trivial configuration of the field ϕ is localized in the finite region R on a spacelike in $g_{\mu\nu}$ hypersurface Σ, and beyond this region the field ϕ is in its vacuum state $\phi = const$, then the front of the solution always propagates with the speed of light. The blue lines correspond to the light rays. The pink cones represent the influence cones for k-*essence*. On the boundary of R the influence cones are equal to light cones.

If we abandon the condition of the regularity of the emergent geometry $G^{-1}_{\mu\nu}$, but still require that the Lagrangian is analytic function of X in the neighborhood of $X = 0$, then the speed of propagation in vacuum is always *smaller* than the speed of light. Indeed in this case the speed of

[4] In particular, it was required in [57] that for models allowing topological k-*defects*, the asymptotic behavior near the trivial vacuum $X = 0$ (at the spatial infinity) is of the form (7.1)

sound c_s is:

$$c_s^2 = \frac{1}{(1 + 2(n-1))} < 1,$$

where n is the power of the first non-zero kinetic term in (7.1).

To demonstrate explicitly the points stated above we will find now exact solitonic solutions in the purely kinetic k-*essence* theories with Lagrangian $\mathcal{L}(X)$ and verify that these solitons propagate in the Minkowski spacetime with the speed of light. Assuming that the scalar field depends only on $\theta \equiv x + vt$ and substituting $\phi = \varphi(\theta)$ in equation (2.3) we find that this equation reduces to

$$\mathcal{L}_{,X}\varphi_{,\theta\theta}\left(v^2 - 1\right) + \mathcal{L}_{,XX}\varphi_{,\theta\theta}\varphi_{,\theta}^2\left(v^2 - 1\right)^2 = 0, \tag{7.2}$$

This equation is trivially satisfied for $v = \pm 1$, that is, there exist solitary waves $\varphi(x \pm t)$ propagating with the speed of light. They are solutions corresponding to rather special initial conditions $\phi_0(x) = \varphi(x)$ and $\dot{\phi}_0(x) = \pm\varphi(x)$. Note that the general solutions are not a superposition of these solitonic solutions because the equation of motion is nonlinear. Assuming that $v \neq \pm 1$ we find that (7.2) is satisfied by either nonlocalized solution $\phi = x \pm vt + const$, or it reduces to:

$$\mathcal{L}_{,X} + \mathcal{L}_{,XX}\varphi_{,\theta}^2\left(v^2 - 1\right) = \mathcal{L}_{,X} + 2X\mathcal{L}_{,XX} = 0, \tag{7.3}$$

This algebraic equation is trivially satisfied for all X if $\mathcal{L} = f(\phi)\sqrt{X} - V(\phi)$. This is the case when the perturbations propagate with the infinite speed on any background [63]. For more general Lagrangians $\mathcal{L}(X)$ equation (7.3) can be solved algebraically to obtain a particular $X_0 = \frac{1}{2}\varphi_{,\theta}^2\left(v^2 - 1\right) = const$. The only solutions of this last equation are either $\varphi(x \pm t)$ or trivial solutions $\phi_{\text{triv}} = const$. For the Born-Infeld Lagrangian [56] the exact solutions of this type were found in [61] (see also [62]). For more complicated Lagrangian, for example, of the form $\mathcal{L} = \mathcal{K}(X) + V(\phi)$ there exist solitonic solutions with $v < 1$ [57].

Thus, we have shown that under reasonable restrictions on the theory the field configurations localized in trivial vacuum never propagate faster than light. Therefore the causal limit for these localized configurations is always governed by the usual gravitational metric.

8. Discussion

In this paper we have considered the k-*essence*-like scalar fields with the Lorentz invariant action (2.1) and have studied the issues of causality and Cauchy problem for such theories. These questions are non-trivial because small perturbations π on backgrounds ϕ_0 can propagate faster-than-light. The perturbations "feel" the effective metric, $G^{\mu\nu}$ given by (2.12), which is different from the gravitational metric $g^{\mu\nu}$, if the Lagrangian $\mathcal{L}$ is a non-linear function of X and the background is nontrivial $\partial_\mu\phi_0 \neq 0$. We have derived the action for the perturbations on an arbitrary background and have shown that these perturbations "feel" the emergent geometry $G^{-1}_{\mu\nu}$. The influence cone determined by $G^{-1}_{\mu\nu}$ is larger than those one determined by metric $g_{\mu\nu}$ provided $\mathcal{L}_{,XX}/\mathcal{L}_{,X} < 0$ [6, 36, 33]. Thus perturbations can propagate with the speed exceeding the speed of light. In this case the background serves as a new *aether* and preselects the preferred reference frame. This is why the causal paradoxes arising in the presence of *tachyons* [5] (superluminal particles in the Minkowski vacuum) do not appear here. In particular, we have shown that in physically interesting situations, namely, cosmological solutions and for the case of a black hole surrounded by an accreting fluid, the closed timelike curves are absent and hence we cannot send the signal to our own past using the superluminal signals build out of the "superluminal" scalar field perturbations. Thus, the k-*essence* models, which generically possess the superluminal propagation, *do not lead to the causal paradoxes*, contrary to the claim in [2, 30].

[5] Do not confuse them with field theoretical tachyons with $m^2 < 0$.

We have shown how to pose correctly the Cauchy problem for the k-*essence* fields with superluminal propagation, which sometimes might seem problematic [8]. The correct initial Cauchy hypersurface Σ must simultaneously be spacelike with respect to both gravitational metric $g_{\mu\nu}$ and the effective metric $G^{-1}_{\mu\nu}$. Because the effective metric $G^{-1}_{\mu\nu}$ itself depends on the values of the field ϕ and its first derivatives, the initial value problem must be set up in a self-consistent manner: in addition to the usually assumed hyperbolicity condition (2.5), one must require that the field ϕ and its derivative on Σ must satisfy the inequality (4.3). In particular, in the case of spacecraft which has very large velocity with respect to the homogeneous background of the k-*essence*, the latter conditions are violated on the hypersurface of constant astronaut proper time. Therefore no physical devices are able to produce an arbitrary configuration of perturbations on this hyperspace.

It was found in [8] that in the theories under consideration one can have the backgrounds possessing the closed causal curves (CCCs). However, as we have argued above, this is not directly related to the superluminal propagation. In fact, the situation here is very similar to the situation in General Relativity, where one can also have the manifolds with the closed causal curves although the speed of propagation is always limited by the speed of light. In this respect the situation in the theories with the superluminal propagation is not worse than in General Relativity. To avoid causal paradoxes in General Relativity, Hawking suggested the *Chronology Protection Conjecture*, which states that the quantum effects and, in particular, vacuum polarization effects can prevent the formation of the closed timelike curves [20]. Similarly to Hawking one may argue that in the case of the superluminal propagation the *Chronology Protection Conjecture* can be valid as well. In fact, this conjecture was already invoked to exclude the causality violation in the case of two pairs of Casimir plates [19]. Once we employ the *Chronology Protection Conjecture*, no constructions admitting CCCs, similar to those presented in [8], are possible.

Sometimes the "superluminal" theories are criticized in the literature on the basis of general, or better to say, aesthetic grounds. For example, Ref. [3] claims: "The spacetime metric is preferred in terms of clock measurements and free fall (geodesic) motion (including light rays), thus underlying General Relativity's central theme of gravity being encoded in spacetime curvature." Although this argument is not more than the matter of taste, we would rather prefer to have General Relativity as a theory which keeps its (restricted) universal meaning even in the presence of superluminal propagation. We have argued that under physically reasonable assumptions and without a preexisting nontrivial background the causality is governed by metric $g_{\mu\nu}$. Indeed, if initially the field ϕ is localized within some finite region of space surrounded by vacuum, then the border of this region propagates with the speed of light and it is impossible to send signals faster than light.

Acknowledgments

We are very thankful to Camille Bonvin, Chiara Caprini, Sergei Dubovsky, Ruth Durrer, Valery Frolov, Robert Helling, Mattew Kleban, Lev Kofman, Stefano Liberati, Alan Rendall, Sergei Sibiryakov, Ilya Shapiro, Alexey Starobinsky, Leonard Susskind, Matt Visser, and especially Sergei Winitzki for very useful discussions. It is a pleasure to thank Sergei Winitzki for helpful comments on the first version of the manuscript. E.B. thanks Alexander von Humboldt foundation and INFN for support. A.V. would like to thank the theory group of Laboratori Nazionali del Gran Sasso, INFN, and organizers and staff of Les Houches Summer School for hospitality during the preparation of this manuscript and during earlier stages of this project respectively.

A. Characteristics and superluminal propagation

Let us consider scalar field ϕ interacting with external source $J(x)$. The equation of motion for the

scalar field is

$$\tilde{G}^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\phi+\varepsilon_{,\phi}=J \tag{A.1}$$

where metric $\tilde{G}^{\mu\nu}$ is given by (2.4) and for brevity we use the "hydrodynamic" notation $\varepsilon\left(X,\phi\right)=2X\mathcal{L}_{,X}-\mathcal{L}$ (see Appendix D). Suppose ϕ_0 is the background solution of (A.1) in the presence of source $J_0\left(x\right)$ and gravitational metric $g_{\mu\nu}\left(x\right)$. Let us consider a slightly perturbed solution $\phi=\phi_0+\pi$ of (A.1) with the source $J=J_0+\delta J$ and the original unperturbed metric $g_{\mu\nu}\left(x\right)$. The equation of motion for π is then

$$\tilde{G}^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\pi+\varepsilon_{,\phi\phi}\pi+\varepsilon_{,\phi X}\delta X+\delta\tilde{G}^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\phi_0=\delta J, \tag{A.2}$$

where

$$\delta X=\nabla_{\nu}\phi_0\nabla^{\nu}\pi\quad\text{ and }\;\delta\tilde{G}^{\mu\nu}=\frac{\partial\tilde{G}^{\mu\nu}}{\partial\phi}\pi+\frac{\partial\tilde{G}^{\mu\nu}}{\partial\nabla_{\alpha}\phi}\nabla_{\alpha}\pi.$$

This equation can be written as

$$\tilde{G}^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\pi+V^{\mu}\nabla_{\mu}\pi+\tilde{M}^2\pi=\delta J, \tag{A.3}$$

where

$$V^{\mu}\left(x\right)\equiv\frac{\partial\tilde{G}^{\alpha\beta}}{\partial\nabla_{\mu}\phi}\nabla_{\alpha}\nabla_{\beta}\phi_0+\varepsilon_{,\phi X}\nabla^{\mu}\phi_0, \tag{A.4}$$

and

$$\tilde{M}^2\left(x\right)\equiv\frac{\partial\tilde{G}^{\alpha\beta}}{\partial\phi}\nabla_{\alpha}\nabla_{\beta}\phi_0+\varepsilon_{,\phi\phi}. \tag{A.5}$$

Considering the eikonal (or short wavelength) approximation [53] we have $\pi\left(x\right)=A\left(x\right)\exp i\omega S\left(x\right)$, where ω is a large dimensionless parameter and the amplitude $A\left(x\right)$ is a slowly varying function. In the limit $\omega\to\infty$ the terms containing no second derivatives, $V^{\mu}\left(x\right)\nabla_{\mu}\pi$ and $\tilde{M}^2\left(x\right)\pi$, become unimportant and (A.3) becomes

$$\tilde{G}^{\mu\nu}\partial_{\mu}S\partial_{\nu}S=0. \tag{A.6}$$

The equation of motion in the eikonal approximation (A.6) is conformally invariant. The surfaces of constant eikonal S (constant phase) correspond to the wave front (characteristic surface) in spacetime. Thus the 1-form $\partial_{\mu}S$ is orthogonal to the characteristic surface. The influence cone at point P is formed by the propagation vectors N^{μ} tangential to the characteristic surface $N^{\mu}\partial_{\mu}S=0$ and positive projection on the time direction. Using (A.6) one can chose $N^{\mu}=\tilde{G}^{\mu\nu}\partial_{\nu}S$ and verify that this vectors are tangential to the characteristic surface. The metric $\tilde{G}^{\mu\nu}$ has an inverse $\tilde{G}^{-1}_{\mu\nu}$ due to the requirement of hyperbolicity (Lorentzian signature of $\tilde{G}^{\mu\nu}$). Therefore $\partial_{\nu}S=\tilde{G}^{-1}_{\mu\nu}N^{\mu}$ and we obtain the equation for the influence cone in the form

$$\tilde{G}^{-1}_{\mu\nu}N^{\mu}N^{\nu}=0.$$

Thus the metric $\tilde{G}^{-1}_{\mu\nu}$ governs the division of acoustic spacetime into past, future and inaccessible "spacelike" regions (or in other words this metric yields the notion of causality). It is well known that this division is invariant under conformal transformations. From action (2.10) for perturbations π, which we derive in Appendix B, it follows that in four dimensions it is natural to consider a conformally transformed metric $G^{-1}_{\mu\nu}=\left(\mathcal{L}^2_{,X}/c_s\right)\tilde{G}^{-1}_{\mu\nu}$. Using this metric from (2.12) one obtains

$$G^{-1}_{\mu\nu}N^{\mu}N^{\nu}=\frac{\mathcal{L}_{,X}}{c_s}g_{\mu\nu}N^{\mu}N^{\nu}-c_s\mathcal{L}_{,XX}\left(\nabla_{\mu}\phi N^{\mu}\right)^2.$$

Therefore

$$g_{\mu\nu}N^{\mu}N^{\nu}=c_s^2\left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)\left(\nabla_{\mu}\phi N^{\mu}\right)^2,$$

and if $\mathcal{L}_{,XX}/\mathcal{L}_{,X}$ is negative, then $g_{\mu\nu}N^{\mu}N^{\nu}<0$, that is, N^{μ} is spacelike and the cone of influence on this background is larger than the light cone: the wave front (or signal) velocity is larger then the speed of light. Note that this is a coordinate independent statement.

B. Action for perturbations

Here we sketch the derivation of action (2.10) for π in the spacetime of arbitrary dimension $N > 2$. First of all we would like to investigate whether there exists a metric $G^{\mu\nu}$ for which the equation of motion for perturbations π takes a canonical (Klein-Gordon) form

$$G^{\mu\nu}D_\mu D_\nu\pi + M_{\text{eff}}^2\pi = \delta I, \tag{B.1}$$

where D_μ is a covariant derivative with associated with the new metric $G^{\mu\nu}$: $D_\mu G^{\alpha\beta} = 0$. Note that the equations of motion (A.2) and (B.1) should have the same influence cone structure. Thus the metrics $G^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ must be related by conformal transformation and if it is really possible to rewrite (A.2) in canonical form, then there must exist $\Omega\left(\phi_0, X_0\right)$, such that

$$G^{\mu\nu} = \Omega\tilde{G}^{\mu\nu}. \tag{B.2}$$

Therefore our first task is to find $\Omega\left(\phi_0, X_0\right)$. Note that this method makes sense for the dimensions $D > 2$ only. That happens because in $D = 2$ all metrics are conformally equivalent to $\eta_{\mu\nu}$ and the wave equation is conformally invariant, see e.g. Ref. [39], P. 447. Let us define the following covariant derivative

$$D_\mu A_\nu = \nabla_\mu A_\nu - L^\lambda_{\mu\nu}A_\lambda \tag{B.3}$$

which is compatible with the new metric whereas $\nabla_\mu A_\nu = \partial_\mu A_\nu - \Gamma^\lambda_{\mu\nu}A_\lambda$ denotes the standard covariant derivative associated with the gravitational metric: $\nabla_\mu g^{\alpha\beta} = 0$, as usual. Note, that the tensor $L^\lambda_{\mu\nu}$ introduced in (B.3) is the difference of the Christoffel symbols corresponding to the effective and gravitational metrics. Comparing (A.2) and (B.1) we infer that

$$\Omega\tilde{G}^{\mu\nu}D_\mu D_\nu\pi + M_{\text{eff}}^2\pi = \Omega\tilde{G}^{\mu\nu}\nabla_\mu\nabla_\nu\pi - \Omega\tilde{G}^{\mu\nu}L^\lambda_{\mu\nu}\nabla_\lambda\pi + M_{\text{eff}}^2\pi$$

must be equal (up to a multiplication by a scalar function Ω) to the l.h.s of (A.3). These can be true only if the following condition holds

$$\tilde{G}^{\mu\nu}L^\lambda_{\mu\nu} = -V^\lambda, \tag{B.4}$$

where V^λ is defined in (A.4). When this condition is satisfied we can always make the redefinition

$$M_{\text{eff}}^2 = \Omega\tilde{M}^2 \quad \text{and} \quad \delta I = \Omega\delta J,$$

where $\tilde{M}^2$ is defined in (A.5). The connection $L^\lambda_{\mu\nu}$ depends on the unknown function Ω (and its derivatives) which has to be obtained form (B.4). To solve (B.4) it is convenient to multiply its both sides by Ω. Then using (A.4) and (B.2) this condition takes the form:

$$G^{\mu\nu}L^\lambda_{\mu\nu} = -\Omega\left(\frac{\partial\tilde{G}^{\alpha\beta}}{\partial\nabla_\lambda\phi}\nabla_\alpha\nabla_\beta\phi_0 + \varepsilon_{,\phi X}\nabla^\lambda\phi_0\right). \tag{B.5}$$

Let us now solve (B.5) with respect to Ω. In complete analogy with the formula (86,6) from Ref. [53] we have

$$G^{\mu\nu}L^\lambda_{\mu\nu} = -\frac{1}{\sqrt{-G}}\nabla_\alpha\left(\sqrt{-G}G^{\alpha\lambda}\right), \tag{B.6}$$

where $\sqrt{-G} = \sqrt{-\text{det}G^{-1}_{\mu\nu}} = \Omega^{-D/2}\sqrt{-\text{det}\tilde{G}^{-1}_{\alpha\beta}}$, and D is the number of dimensions of the spacetime. Using the formula (B14) from Ref. [33] one obtains

$$\text{det}\tilde{G}^{\alpha\beta} = \left(\mathcal{L}_{,X}\right)^D c_s^{-2}\text{det}\left(g^{\mu\nu}\right), \text{ and } \text{det}\tilde{G}^{-1}_{\alpha\beta} = \left(\mathcal{L}_{,X}\right)^{-D} c_s^2\text{det}\left(g_{\mu\nu}\right). \tag{B.7}$$

Finally we arrive to the relation,

$$\sqrt{-G} = c_s \sqrt{-g} \left(\Omega \mathcal{L}_{,X}\right)^{-D/2}. \tag{B.8}$$

It is convenient to introduce the auxiliary function

$$F = c_s \left(\Omega \mathcal{L}_{,X}\right)^{-D/2} \Omega. \tag{B.9}$$

and then using (B.6), we can rewrite equation (B.5) as:

$$\nabla_\alpha \left(F \tilde{G}^{\alpha\lambda}\right) = F \left(\frac{\partial \tilde{G}^{\alpha\beta}}{\partial \nabla_\lambda \phi} \nabla_\alpha \nabla_\beta \phi_0 + \varepsilon_{,\phi X} \nabla^\lambda \phi_0 \right). \tag{B.10}$$

Differentiating the metric $\tilde{G}^{\alpha\lambda}$ from the l.h.s. of the last equation in accordance with the chain rule we find:

$$\tilde{G}^{\alpha\lambda} \nabla_\alpha F = F \left(\left(\frac{\partial \tilde{G}^{\alpha\beta}}{\partial \nabla_\lambda \phi} - \frac{\partial \tilde{G}^{\alpha\lambda}}{\partial \nabla_\beta \phi} \right) \nabla_\alpha \nabla_\beta \phi_0 - \left(\frac{\partial \tilde{G}^{\alpha\lambda}}{\partial \phi} - \varepsilon_{,\phi X} g^{\lambda\alpha} \right) \nabla_\alpha \phi_0 \right). \tag{B.11}$$

Further we obtain

$$\frac{\partial \tilde{G}^{\alpha\lambda}}{\partial \phi} \nabla_\alpha \phi_0 = \left(\mathcal{L}_{,X\phi} + 2X \mathcal{L}_{,XX\phi}\right) \nabla^\lambda \phi_0 = \varepsilon_{,\phi X} \nabla^\lambda \phi_0. \tag{B.12}$$

For the first term in the brackets in (B.11) we have:

$$\frac{\partial \tilde{G}^{\alpha\beta}}{\partial \nabla_\lambda \phi} = \mathcal{L}_{,XX} \left(g^{\alpha\beta} \nabla^\lambda \phi_0 + g^{\lambda\alpha} \nabla^\beta \phi_0 + g^{\lambda\beta} \nabla^\alpha \phi_0 \right) + \mathcal{L}_{,XXX} \nabla^\alpha \phi_0 \nabla^\beta \phi_0 \nabla^\lambda \phi_0, \tag{B.13}$$

and therefore

$$\frac{\partial \tilde{G}^{\alpha\beta}}{\partial \nabla_\lambda \phi} - \frac{\partial \tilde{G}^{\alpha\lambda}}{\partial \nabla_\beta \phi} = 0. \tag{B.14}$$

Thus the r.h.s. of (B.11) identically vanishes. Note that there exists the inverse matrix $\tilde{G}^{-1}_{\alpha\lambda}$ to $\tilde{G}^{\alpha\lambda}$. Therefore from (B.11) we conclude that $\nabla_\alpha F = 0$ or $F = const$ on all backgrounds and for all theories. Considering the linear case, $\mathcal{L}(\phi, X) = X - V(\phi)$, we infer that $F = c_s \left(\Omega \mathcal{L}_{,X}\right)^{-D/2} \Omega = 1$ or

$$\Omega = \left(c_s \mathcal{L}_{,X}^{-D/2} \right)^{1/(D/2-1)}. \tag{B.15}$$

Having calculated Ω we can formulate the main result of this Appendix as follows: the action from which one can obtain the equation of motion in the canonical Klein-Gordon form (B.1) is

$$S_\pi = \frac{1}{2} \int d^D x \sqrt{-G} \left[G^{\mu\nu} \partial_\mu \pi \partial_\nu \pi - M_{\text{eff}}^2 \pi^2 + 2\pi \delta I \right], \tag{B.16}$$

where the emergent metric $G^{\mu\nu}$ is the conformally transformed eikonal metric $\tilde{G}^{\mu\nu}$, defined in (2.4),

$$G^{\mu\nu} \equiv \left(c_s \mathcal{L}_{,X}^{-D/2} \right)^{1/(D/2-1)} \tilde{G}^{\mu\nu} = \left(\frac{c_s}{\mathcal{L}_{,X}} \right)^{1/(D/2-1)} \left[g^{\mu\nu} + \left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} \right) \nabla^\mu \phi \nabla^\nu \phi \right]. \tag{B.17}$$

The inverse metric $G^{-1}_{\mu\nu}$ can be easily calculated using the ansatz $G^{-1}_{\mu\nu} = \alpha g_{\mu\nu} + \beta \nabla_\mu \phi_0 \nabla_\nu \phi_0$ and is given by the formula

$$G^{-1}_{\mu\nu} = \left(\frac{c_s}{\mathcal{L}_{,X}} \right)^{-1/(D/2-1)} \left[g^{\mu\nu} - c_s^2 \left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} \right) \nabla^\mu \phi_0 \nabla^\nu \phi_0 \right]. \tag{B.18}$$

Finally the effective mass is

$$M_{\mathrm{eff}}^2=\left(c_s\mathcal{L}_{,X}^{-N/2}\right)^{1/(D/2-1)}\left[2X\mathcal{L}_{,X\phi\phi}-\mathcal{L}_{,\phi\phi}+\frac{\partial\tilde{G}^{\mu\nu}}{\partial\phi}\nabla_\mu\nabla_\nu\phi_0\right], \tag{B.19}$$

and the effective source for perturbations is given by

$$\delta I=\left(c_s\mathcal{L}_{,X}^{-D/2}\right)^{1/(D/2-1)}\delta J. \tag{B.20}$$

For the reference we also list the formula

$$\sqrt{-G}=\sqrt{-g}\left(\frac{\mathcal{L}_{,X}^D}{c_s^2}\right)^{1/(D-2)}. \tag{B.21}$$

C. Action for the cosmological perturbations

Here we compare the action (2.10) with the action for scalar cosmological perturbations from Refs. [34, 28]. In particular we show that cosmological perturbations propagate in the metric (2.12) but have an effective mass different from (2.9). Finally we derive the generally covariant action for the scalar cosmological perturbations.

To begin with let us consider the action (2.10) for a perturbations $\pi\left(\eta,\mathbf{x}\right)$ around a homogeneous background $\phi(\eta)$ in the spatially flat Friedmann universe

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=a^2\left(\eta\right)\left(d\eta^2-d\mathbf{x}^2\right)=a^2\left(\eta\right)\eta_{\mu\nu}dx^\mu dx^\nu \tag{C.1}$$

where η is the conformal time $\eta=\int dt/a\left(t\right)$ and $\eta_{\mu\nu}$ is the standard Minkowski metric. Using Eq. (2.6) and Eq. (C.1) one can calculate the effective line element (2.13):

$$\begin{aligned} dS^2 &= G_{\mu\nu}^{-1}dx^\mu dx^\nu=\frac{\mathcal{L}_{,X}}{c_s}\left[ds^2-a^2c_s^2\left(\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)2Xd\eta^2\right]= \\ &= \frac{\mathcal{L}_{,X}}{c_s}a^2\left(c_s^2d\eta^2-d\mathbf{x}^2\right)\equiv c_sA^2\left(c_s^2d\eta^2-d\mathbf{x}^2\right). \end{aligned} \tag{C.2}$$

where we have introduced the convenient variable

$$A\equiv\sqrt{\varepsilon_{,X}}a. \tag{C.3}$$

Note that for the models respecting the NEC ($\mathcal{L}_{,X}\geq 0$) the hyperbolicity condition (2.5) requires $\varepsilon_{,X}>0$ and therefore A is always well defined. The factor $\sqrt{-G}$ can be then calculated either from the last expression above (C.2) or from the general expression (B.21):

$$\sqrt{-G}=\frac{\mathcal{L}_{,X}^2}{c_s}a^4=c_s^3A^4. \tag{C.4}$$

Using the formulas (2.8) and (2.6) we calculate the kinetic term

$$G^{\mu\nu}\partial_\mu\pi\partial_\nu\pi=\left(c_sa^2\mathcal{L}_{,X}\right)^{-1}\left(\left(\pi'\right)^2-c_s^2(\vec{\nabla}\pi)^2\right). \tag{C.5}$$

Thus in the case when the perturbations π do not influence the metric $g_{\mu\nu}$ the action (2.10) takes the form

$$S_\pi=\frac{1}{2}\int d^3xd\eta\left[a^2\varepsilon_{,X}\left(\left(\pi'\right)^2-c_s^2(\vec{\nabla}\pi)^2\right)-M_{\mathrm{eff}}^2\frac{\mathcal{L}_{,X}^2}{c_s}a^4\pi^2\right], \tag{C.6}$$

here we have used the definitions of the sound speed (2.6) and energy density (D.2). It is convenient to introduce the canonical normalization for the perturbations. This is achieved by the following field redefinition:

$$\nu = \sqrt{\varepsilon_{,X}}a\pi = \pi A. \tag{C.7}$$

Finally integrating by parts and dropping the total derivative terms we obtain the following "canonical" action

$$S_\pi = \frac{1}{2}\int d^3x d\eta \left[(\nu')^2 - c_s^2(\vec{\nabla}\nu)^2 - m_{\text{eff}}^2\nu^2\right] \tag{C.8}$$

where the new effective mass m_{eff} is given by the following expression

$$m_{\text{eff}}^2 = M_{\text{eff}}^2\frac{\sqrt{-G}}{A^2} - \frac{A''}{A} = \frac{a^2}{\varepsilon_{,X}}\left[\varepsilon_{,\phi\phi} + \frac{\partial\tilde{G}^{\mu\nu}}{\partial\phi}\nabla_\mu\nabla_\nu\phi_0\right] - \frac{\left(\sqrt{\varepsilon_{,X}}a\right)''}{\sqrt{\varepsilon_{,X}}a}. \tag{C.9}$$

or in other terms

$$m_{\text{eff}}^2 = \frac{1}{\varepsilon_{,X}}\left[\varepsilon_{,X\phi}\phi'' + \mathcal{H}\phi'\left(3p_{,X\phi} - \varepsilon_{,X\phi}\right) + \varepsilon_{,\phi\phi}a^2\right] - \frac{\left(\sqrt{\varepsilon_{,X}}a\right)''}{\sqrt{\varepsilon_{,X}}a}. \tag{C.10}$$

Now let us consider the case of cosmological perturbations in the case where the field ϕ is responsible for the dynamics of the Friedmann universe. Following [28, 34] one introduces a canonical variable υ

$$\upsilon \equiv \sqrt{\varepsilon_{,X}}a\left(\delta\phi + \frac{\phi'}{\mathcal{H}}\Psi\right) = A\left(\delta\phi + \frac{\phi'}{\mathcal{H}}\Psi\right), \tag{C.11}$$

and a convenient auxiliary variable z

$$z \equiv \frac{\phi'}{\mathcal{H}}\sqrt{\varepsilon_{,X}}a = \frac{\phi'}{\mathcal{H}}A, \tag{C.12}$$

where $\delta\phi$ is the gauge invariant perturbation of the scalar field, $\mathcal{H} \equiv a'/a$ and $\Psi = \Phi$ is the gauge invariant Newtonian potential. Using this notation the action for scalar cosmological perturbations takes the form:

$$S_{\text{cosm}} = \frac{1}{2}\int d^3x d\eta \left[(\upsilon')^2 - c_s^2(\vec{\nabla}\upsilon)^2 - m_{\text{cosm}}^2\upsilon^2\right] \tag{C.13}$$

where

$$m_{\text{cosm}}^2 \equiv -\frac{z''}{z}. \tag{C.14}$$

It is easily to check that for all cases besides canonical field without potential $\mathcal{L}(\phi, X) \equiv X$

$$m_{\text{cosm}}^2 \neq m_{\text{eff}}^2. \tag{C.15}$$

However, comparing the action (C.8) with C.13 one arrives to conclusion that the cosmological perturbations propagate in the same metric (2.8), (2.12). Further one can introduce the notation $\overline{\delta\phi}$ for the sometimes so-called "scalar perturbations on the spatially flat slicing"

$$\overline{\delta\phi} \equiv \delta\phi + \frac{\phi'}{\mathcal{H}}\Psi. \tag{C.16}$$

For this scalar field the action for cosmological perturbations (C.13) takes the form

$$S_{\text{cosm}} = \frac{1}{2}\int d^4x\sqrt{-G}\left[G^{\mu\nu}\partial_\mu\overline{\delta\phi}\partial_\nu\overline{\delta\phi} - M_{\text{cosm}}^2\overline{\delta\phi}^2\right], \tag{C.17}$$

thus the cosmological perturbations $\overline{\delta\phi}$ live in the emergent acoustic spacetime with the metric (2.8), (2.12). Similarly as we have calculated in (C.9) we have

$$M_{\text{cosm}}^2\frac{\sqrt{-G}}{A^2} - \frac{A''}{A} = M_{\text{cosm}}^2 a^2\mathcal{L}_{,X}c_s - \frac{\left(\sqrt{\varepsilon_{,X}}a\right)''}{\sqrt{\varepsilon_{,X}}a} = -\frac{z''}{z} \tag{C.18}$$

after some algebra the last expression reduces to

$$\chi'' + 2\left(\frac{A'}{A}\right)\chi' + A^2\left(M^2_{\mathrm{cosm}} c_s^3\right)\chi = 0 \tag{C.19}$$

where we have introduced a new auxiliary field

$$\chi(\eta) \equiv \frac{\phi'}{\mathcal{H}} = \left(\frac{3}{8\pi G_N}\right)^{1/2}\sqrt{\frac{2X}{\varepsilon}}. \tag{C.20}$$

The equation (C.19) is in turn the Klein-Gordon equation

$$\left(\Box_{\overline{g}} + \left(M^2_{\mathrm{cosm}} c_s^3\right)\right)\chi = 0 \tag{C.21}$$

for the field χ in the metric $\overline{g}_{\mu\nu} \equiv A^2\eta_{\mu\nu} = \varepsilon_{,X} g_{\mu\nu}$ conformally related to the gravitational metric $g_{\mu\nu}$. Thus we have

$$M^2_{\mathrm{cosm}} = -c_s^{-3}\chi^{-1}\Box_{\overline{g}}\chi. \tag{C.22}$$

One can rewrite this formula in terms of the gravitational metric $g_{\mu\nu}$. Using the rules of the conformal transformations we have

$$\Box_{\overline{g}}\chi = \frac{1}{\sqrt{-\overline{g}}}\nabla_\mu\left(\sqrt{-\overline{g}}\overline{g}^{\mu\nu}\nabla_\nu\chi\right) = \frac{1}{\varepsilon^2_{,X}}\frac{1}{\sqrt{-g}}\nabla_\mu\left(\varepsilon_{,X}\sqrt{-g}g^{\mu\nu}\nabla_\nu\chi\right) = \tag{C.23}$$

$$= -\nabla^\mu\chi\nabla_\mu\varepsilon^{-1}_{,X} + \varepsilon^{-1}_{,X}\Box_g\chi \tag{C.24}$$

Thus the effective mass for cosmological perturbations $\overline{\delta\phi}$ is

$$M^2_{\mathrm{cosm}} = -c_s^{-3}\varepsilon^{-1}_{,X}\left(\sqrt{\frac{\varepsilon}{X}}\Box_g\sqrt{\frac{X}{\varepsilon}} + \nabla_\mu\ln\left(\varepsilon_{,X}\right)\nabla^\mu\ln\sqrt{\frac{X}{\varepsilon}}\right). \tag{C.25}$$

Note that in the case of canonical kinetic terms $\mathcal{L}(\phi, X) = X - V(\phi)$ the last expression for M^2_{cosm} the simplifies to

$$M^2_{\mathrm{cosm,canonical}} = -\left(w+1\right)^{-1/2}\Box_g\left(w+1\right)^{1/2}. \tag{C.26}$$

where $w = p/\varepsilon$ is the equation of state parameter. In particular for the universe filled with the massless canonical scalar field $M_{\mathrm{cosm},} = 0$.

D. Effective Hydrodynamics

It is well-known that for timelike $\nabla_\nu\phi$ ($X > 0$ in our signature) one can employ the hydrodynamic approach to describe the system with the action (2.1). To do this one need to introduce a four-velocity as follows:

$$u_\mu \equiv \frac{\nabla_\mu\phi}{\sqrt{2X}}. \tag{D.1}$$

Using (D.1) the energy momentum tensor (2.2) tensor can be rewritten in the perfect fluid form:

$$T_{\mu\nu} = \left(\varepsilon + p\right)u_\mu u_\nu - p g_{\mu\nu},$$

where the pressure coincides with the Lagrangian density, $p = \mathcal{L}(X, \phi)$, and the energy density is

$$\varepsilon\left(X, \phi\right) = 2Xp_{,X} - p. \tag{D.2}$$

The sound speed (2.6) can be expressed [28] as

$$c_s^2 = \frac{p_{,X}}{\varepsilon_{,X}} = \left(\frac{\partial p}{\partial\varepsilon}\right)_\phi. \tag{D.3}$$

In what follows we restrict ourselves to the class of Lagrangians which do not depend of ϕ explicitly, $p = p(X)$ and in addition we require that $X > 0$. This class of models is equivalent to perfect fluid models with zero vorticity and with the pressure being a function of the energy density only, $p = p(\epsilon)$. Then the expressions (2.6) or (D.3) coincide with the usual definition of the sound speed for the perfect fluid: $c_s^2 = \partial p/\partial\varepsilon$. Apart from the energy density ε and pressure p one can also formally introduce the "concentration of particles":

$$n \equiv \exp\left(\int \frac{d\varepsilon}{\varepsilon + p(\varepsilon)}\right) = \sqrt{X}p_{,X}.$$

and the enthalpy

$$h \equiv \frac{\varepsilon + p}{n} = 2\sqrt{X}.$$

In particular the equation of motion (2.3) takes the form of the particle number conservation law: $\nabla_\mu (nu^\mu) = 0$. Using these definitions we can rewrite the induced metric metric $G^{\mu\nu}$ and its inverse in terms of hydrodynamic quantities only:

$$G^{\mu\nu} = \frac{hc_s}{2n}\left[g^{\mu\nu} - \left(1 - c_s^{-2}\right)u^\mu u^\nu\right], \tag{D.4}$$

$$G^{-1}_{\mu\nu} = \frac{2n}{hc_s}\left[g_{\mu\nu} - \left(1 - c_s^2\right)u_\mu u_\nu\right]. \tag{D.5}$$

To our best knowledge these metrics (D.4) along with an action for the velocity potentials were introduced for the first time in [55], where the accretion of the perfect fluid onto black hole was studied. As it follows from the derivation in Appendix B, the metric (B.17) and the action (B.16) derived in our paper are applicable in the more general case of arbitrary nonlinear scalar field theories $\mathcal{L}(X,\phi)$ and for all possible (not only timelike $X_0 > 0$) backgrounds produced by any external sources. Note that the scalar field theory with Lagrangian $\mathcal{L}(X,\phi)$, which explicitly depends on ϕ, is not equivalent to the isentropic hydrodynamics, because ϕ and X are independent and therefore the pressure cannot be expressed though ε only.

E. Green functions for a moving spacecraft

Here we calculate the retarded Green's function for a moving spacecraft in the case of three spatial dimensions. First we calculate the retarded Green's function in the preferred (rest) frame and then we perform the Lorentz boost (with the invariant speed c) for the solution. We compare the result with one obtained by the direct calculation of Green's function for the Eq. (5.2). We will need the following formulas (Gradshtein, Ryzhik, p.750):

$$\int_a^\infty J_0\left(b\sqrt{x^2 - a^2}\right)\sin(cx) = \frac{\cos\left(a\sqrt{c^2 - b^2}\right)}{\sqrt{c^2 - b^2}}, \quad \text{for} \quad 0 < b < c \tag{E.1}$$

$$= 0, \quad \text{for} \quad 0 < c < b \tag{E.2}$$

$$\int_a^\infty J_0\left(b\sqrt{x^2 - a^2}\right)\cos(cx) = -\frac{\sin\left(a\sqrt{c^2 - b^2}\right)}{\sqrt{c^2 - b^2}}, \quad \text{for} \quad 0 < b < c \tag{E.3}$$

$$= \frac{\exp\left(-a\sqrt{b^2 - c^2}\right)}{\sqrt{b^2 - c^2}}, \quad \text{for} \quad 0 < c < b \tag{E.4}$$

$$\int_0^a J_0\left(b\sqrt{a^2 - x^2}\right)\cos(cx) = \frac{\sin\left(a\sqrt{c^2 + b^2}\right)}{\sqrt{c^2 + b^2}}, \quad \text{for} \quad 0 < b \tag{E.5}$$

In the preferred frame the Green function is (see e.g. [52])

$$G_R^{\text{rf}}(t,x) = \frac{\theta(t)}{2c_s\pi}\delta\left(c_s^2 t^2 - |x|^2\right). \tag{E.6}$$

Performing the Lorentz transformation $x = \gamma\,(x' + vt')$, $t = \gamma\,(t' + vx')$, where $\gamma = \left(1 - v^2\right)^{-1/2}$ we find the Green function in the moving frame:

$$G_R^{\rm rf}(t', x') = \frac{\theta\,(t' + vx')}{2c_s\pi}\delta\left[\gamma^2\left(c_s^2\,(t' + vx')^2 - (x' + vt')^2\right) - y^2 - z^2\right]. \tag{E.7}$$

We need to calculate the Fourier transform to the function (E.7). It is convenient to shift x' as follows:

$$x' = \overline{x} - vt'\left(\frac{1 - c_s^2}{1 - c_s^2 v^2}\right). \tag{E.8}$$

Then the argument of the delta-function in (E.7) can be rewritten as

$$\gamma^2\left(c_s^2\,(t' + vx')^2 - (x' + vt')^2\right) - y^2 - z^2 = \alpha c_s^2 t'^2 - \alpha^{-1}\overline{x}^2 - y^2 - z^2,$$

where

$$\alpha = \frac{1 - v^2}{1 - c_s^2 v^2}. \tag{E.9}$$

Now we are ready to proceed with the Fourier transform of (E.7):

$$G_R^{\rm rf}(t', k') = \frac{{\rm e}^{i\varphi}}{2c_s\pi}\int_{-\infty}^{\infty} d\overline{x}dydz\,\theta\,(t' + vx')\;\delta\left(\alpha c_s^2 t'^2 - \alpha^{-1}\overline{x}^2 - y^2 - z^2\right){\rm e}^{ik_{x'}\overline{x} + ik_y y + ik_z z} \tag{E.10}$$

where we introduced the notation:

$$\varphi = -k_{x'}vt'\left(\frac{1 - c_s^2}{1 - c_s^2 v^2}\right). \tag{E.11}$$

Step-function in the integral implies that the integration over $\overline{x}$ is made from x_* to $+\infty$:

$$G_R^{\rm rf}(t', k') = \frac{{\rm e}^{i\varphi}}{2c_s\pi}\int_{x_*}^{\infty} d\overline{x}\int_{-\infty}^{\infty} dy\int_{-\infty}^{\infty} dz\,\delta\left(\alpha c_s^2 t'^2 - \alpha^{-1}\overline{x}^2 - y^2 - z^2\right){\rm e}^{ik_{x'}\overline{x} + ik_y y + ik_z z}, \tag{E.12}$$

$$x_* = vt'\left(\frac{1 - c_s^2}{1 - c_s^2 v^2}\right) - \frac{t'}{v} = -\frac{t'}{v}\left(\frac{1 - v^2}{1 - c_s^2 v^2}\right) = -\frac{\alpha}{v}t'. \tag{E.13}$$

Introducing $r \equiv \sqrt{y^2 + z^2}$, ϕ as the angle between the vectors $\{k_y, k_z\}$ and $\{y, z\}$ and $k_\perp \equiv \sqrt{k_y^2 + k_z^2}$ we obtain:

$$G_R^{\rm rf}(t', k') = \frac{{\rm e}^{i\varphi}}{2c_s\pi}\int_{x_*}^{\infty} d\overline{x}\int_0^{\infty} drr\int_0^{2\pi} d\phi\delta\left(\alpha c_s^2 t'^2 - \alpha^{-1}\overline{x}^2 - r^2\right){\rm e}^{ik_{x'}\overline{x} + ik_\perp r\cos\phi}. \tag{E.14}$$

Integrating over r first gives:

$$G_R^{\rm rf}(t', k') = \frac{{\rm e}^{i\varphi}}{4c_s\pi}\int_{x_*}^{+\infty} d\overline{x}\int_0^{2\pi} d\phi\exp\left(ik_{x'}\overline{x} + ik_\perp\sqrt{\alpha c_s^2 t'^2 - \alpha^{-1}\overline{x}^2}\cos\phi\right) \tag{E.15}$$

for

$$\alpha c_s^2 t'^2 - \alpha^{-1}\overline{x}^2 > 0, \tag{E.16}$$

otherwise it is zero. Integrating (E.15) over ϕ we find:

$$G_R^{\rm rf}(t', k') = \frac{{\rm e}^{i\varphi}}{2c_s}\int_{x_*}^{+\infty} d\overline{x}J_0\left(k_\perp\sqrt{\alpha c_s^2 t'^2 - \alpha^{-1}\overline{x}^2}\right)\exp\left(ik_{x'}\overline{x}\right), \tag{E.17}$$

where $J_0(x)$ is the Bessel function of the zeroth order. Now we need to integrate the expression (E.17) taking into account the condition (E.16).We consider two cases separately: the case of slow spacecraft, $v^2c_s^2 < 1$ ($\alpha > 0$), and the case of rapid spacecraft, $v^2c_s^2 > 1$ ($\alpha < 0$).

For the slow spacecraft we easily obtain from (E.17) and (E.16):

$$
\begin{aligned}
G_R^{\rm rf}(t',k') &= \frac{\mathrm{e}^{i\varphi}}{2c_s}\theta(t')\int_{-\alpha c_s t'}^{\alpha c_s t'} d\overline{x}\, J_0\left(k_\perp\sqrt{\alpha c_s^2 t'^2-\alpha^{-1}\overline{x}^2}\right)e^{ik_{x'}\overline{x}} \\
&= \frac{\mathrm{e}^{i\varphi}}{c_s}\theta(t')\int_{0}^{\alpha c_s t'} d\overline{x}\, J_0\left(\frac{k_\perp}{\sqrt{\alpha}}\sqrt{\alpha^2 c_s^2 t'^2-\overline{x}^2}\right)\cos\left(k_{x'}\overline{x}\right).
\end{aligned}
$$

Using (E.5) we then find the Green's function for slow moving spacecraft:

$$
\begin{aligned}
G_R^{\rm rf}(t',k') &= -\theta\left(t'\right)\frac{i\mathrm{e}^{i\varphi}}{2c_s\sqrt{k_{x'}^2+k_\perp^2/\alpha}}\left(\mathrm{e}^{i\alpha c_s t'\sqrt{k_{x'}^2+k_\perp^2/\alpha}}-\mathrm{e}^{-i\alpha c_s t'\sqrt{k_{x'}^2+k_\perp^2/\alpha}}\right) \\
&= \theta\left(t'\right)\frac{1}{2ic_s}\left(k_{x'}^2+k_\perp^2\frac{1-c_s^2v^2}{1-v^2}\right)^{-1/2}\left(\mathrm{e}^{i\omega_+t'}-\mathrm{e}^{i\omega_-t'}\right).
\end{aligned} \tag{E.18}
$$

In the case of rapid spacecraft, $v^2c_s^2>1$ ($\alpha<0$), one can verify that $\alpha^2c_s^2t'^2>x_*^2$ for any t'. Thus (E.17) along with (E.16) can be rewritten as:

$$
G_R^{\rm rf}(t',k') = \frac{\mathrm{e}^{i\varphi}}{2c_s}\int_{|\alpha c_s t'|}^{+\infty} d\overline{x}\, J_0\left(\frac{k_\perp}{\sqrt{|\alpha|}}\sqrt{\alpha^2c_s^2t'^2-\overline{x}^2}\right)\left(\cos\left(k_{x'}\overline{x}\right)+i\sin\left(k_{x'}\overline{x}\right)\right). \tag{E.19}
$$

Using (E.2) and (E.4) for $k_\perp^2>|\alpha|\,k_{x'}^2$ and (E.1) and (E.3) for $k_\perp^2<|\alpha|\,k_{x'}^2$ we obtain in both cases:

$$
G_R^{\rm rf}(t',k') = \frac{\mathrm{e}^{i\varphi}}{2c_s}\frac{\exp\left(-\left|\alpha c_s t'\right|\sqrt{k_\perp^2\,|\alpha|^{-1}-k_{x'}^2}\right)}{\sqrt{k_\perp^2\,|\alpha|^{-1}-k_{x'}^2}}. \tag{E.20}
$$

The last expression can be written as

$$
G_R^{\rm rf}(t',k') = -\frac{1}{2ic_s}\left(k_{x'}^2+k_\perp^2\frac{1-c_s^2v^2}{1-v^2}\right)^{-1/2}\left(\theta\left(t'\right)\mathrm{e}^{i\omega_+t'}+\theta\left(-t'\right)\mathrm{e}^{i\omega_-t'}\right). \tag{E.21}
$$

Thus the modes propagating in with

$$
k_\perp^2 > k_{x'}^2\,|\alpha| = k_{x'}^2\left(\frac{1-v^2}{c_s^2v^2-1}\right) \tag{E.22}
$$

are exponentially suppressed. The singular directions $k_\perp^2=k_{x'}^2\,|\alpha|$ are unphysical because they have measure zero in the integral. This directions correspond to the sufficient but integrable singularities in the Green function.

If the Green's function is calculated directly from the Eq. (5.2) by means of standard approach then one can find, that the solution is:

$$
G_R^{\rm sc}(t',k') = \theta\left(t'\right)\frac{1}{2ic_s}\left(k_{x'}^2+k_\perp^2\frac{1-c_s^2v^2}{1-v^2}\right)^{-1/2}\left(\mathrm{e}^{i\omega_+t'}-\mathrm{e}^{i\omega_-t'}\right), \tag{E.23}
$$

which coincides with the Green's function (E.18) we calculated by applying the Lorentz transformation to the rest Green's function in the case of slow motion. Note, however, that the results differs for the case of fast moving spacecraft - compare (E.23) and (E.21). The function $G_R^{\rm sc}(t',k')$ from (E.23) contains exponentially growing modes for sufficiently high $k_\perp$, while correct way of calculation gave us a sensible result (E.21) - it contains only exponentially suppressed modes. This makes sense because the late time solution approaches the free wave which do not contain these high $k_\perp$.

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